Electronics & Communication Engineering

Formulae Book

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and many more........
Dear Students,

It’s a proud moment for all of us.

When **Electronics & Communication Engineering** is playing pivotal role in deciding the future of our development plans, we have set an ambitious plan to augment your study material with exceptionally well written material to help you learn better.

First and foremost let me discuss about GATE exam. GATE is basically an objective type examination, conducts IITs and IISc in the month of February every year. Now a day’s GATE examination gained lot of importance because, not only for M.Tech admissions but also for Job in PSUs.

Now, let me discuss about Indian Engineering Services abbreviated as IES are the civil services that meet the technical and managerial functions of the Government of India. Large number of candidates takes these exams, competing for limited posts. IES officers are selected by the union government on the recommendations made by the Union Public Service Commission (UPSC).

So, now the question is how to succeed in the above exam? For this exam one need to prepare according to syllabus provided in notification. In this exam, primarily examiners test your fundamental concepts in each and every subject according to their priority. So, one needs to know clearly how to prepare to secure good rank, focussing on this very issue, Vani Institute is providing solutions in this book.

We have decoded to bring out a well crafted gem in the form of this **Formulae book** for enhancing your learning abilities.

Therefore, sincerely hope that this **Formulae book** will help you achieve even greater heights in your Subject and engineering.

We have developed this **Formulae book** according to the best knowledge, in case any errors which might have occurred out of sight, please feel free to inform us your valuable suggestions.

I wish you all very best for your future endeavours.

**Director,**

VANI INSTITUTE
Network solution methods: nodal and mesh analysis; Network theorems: superposition, Thevenin and Norton’s, maximum power transfer; Wye-Delta transformation; Steady state sinusoidal analysis using phasors; Time domain analysis of simple linear circuits; Solution of network equations using Laplace transform; Frequency domain analysis of RLC circuits; Linear 2-port network parameters: driving point and transfer functions; State equations for networks.

Signals and Systems

Continuous-time signals: Fourier series and Fourier transform representations, sampling theorem and applications; Discrete-time signals: discrete-time Fourier transform (DTFT), DFT, FFT, Z-transform, interpolation of discrete-time signals; LTI systems: definition and properties, causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structure, frequency response, group delay, phase delay, digital filter design techniques.

Electronic Devices

Energy bands in intrinsic and extrinsic silicon; Carrier transport: diffusion current, drift current, mobility and resistivity; Generation and recombination of carriers; Poisson and continuity equations; P-N junction, Zener diode, BJT, MOS capacitor, MOSFET, LED, photo diode and solar cell; Integrated circuit fabrication process: oxidation, diffusion, ion implantation, photolithography and twin-tub CMOS process.
Electronics & Communication Engineering

**Analog Circuits**

Small signal equivalent circuits of diodes, BJT and MOSFETs; Simple diode circuits: clipping, clamping and rectifiers; Single-stage BJT and MOSFET amplifiers: biasing, bias stability, mid-frequency small signal analysis and frequency response; BJT and MOSFET amplifiers: multi-stage, differential, feedback, power and operational; Simple op-amp circuits; Active filters; Sinusoidal oscillators: criterion for oscillation, single-transistor and op-amp configurations; Function generators, wave-shaping circuits and 555 timers; Voltage reference circuits; Power supplies: ripple removal and regulation.

**Digital Circuits**

Number systems; Combinatorial circuits: Boolean algebra, minimization of functions using Boolean identities and Karnaugh map, logic gates and their static CMOS implementations, arithmetic circuits, code converters, multiplexers, decoders and PLAs; Sequential circuits: latches and flip-flops, counters, shift-registers and finite state machines; Data converters: sample and hold circuits, ADCs and DACs; Semiconductor memories: ROM, SRAM, DRAM; 8-bit microprocessor (8085): architecture, programming, memory and I/O interfacing.

**Control Systems**

Basic control system components; Feedback principle; Transfer function; Block diagram representation; Signal flow graph; Transient and steady-state analysis of LTI systems; Frequency response; Routh-Hurwitz and Nyquist stability criteria; Bode and root-locus plots; Lag, lead and lag-lead compensation; State variable model and solution of state equation of LTI systems.
**Communications**

Random processes: autocorrelation and power spectral density, properties of white noise, filtering of random signals through LTI systems; Analog communications: amplitude modulation and demodulation, angle modulation and demodulation, spectra of AM and FM, super heterodyne receivers, circuits for analog communications; Information theory: entropy, mutual information and channel capacity theorem; Digital communications: PCM, DPCM, digital modulation schemes, amplitude, phase and frequency shift keying (ASK, PSK, FSK), QAM, MAP and ML decoding, matched filter receiver, calculation of bandwidth, SNR and BER for digital modulation; Fundamentals of error correction, Hamming codes; Timing and frequency synchronization, inter-symbol interference and its mitigation; Basics of TDMA, FDMA and CDMA.

**Electromagnetics**

Electrostatics; Maxwell’s equations: differential and integral forms and their interpretation, boundary conditions, wave equation, Poynting vector; Plane waves and properties: reflection and refraction, polarization, phase and group velocity, propagation through various media, skin depth; Transmission lines: equations, characteristic impedance, impedance matching, impedance transformation, S-parameters, Smith chart; Waveguides: modes, boundary conditions, cut-off frequencies, dispersion relations; Antennas: antenna types, radiation pattern, gain and directivity, return loss, antenna arrays; Basics of radar; Light propagation in optical fibers.
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Electrical Circuits

Formulas
Electron Charge = $1.68 \times 10^{-19}$ C

$I = \frac{dQ}{dt} \text{ C/Sec (or) A}$

$V = \frac{dW}{dQ} \text{ J/C (or) Volts}$

$P = \frac{dW}{dt} \text{ J/Sec (or) Watts}$

$W = \int_{-\infty}^{t} P \, dt \text{ Watt – sec (or) Joules.}$

**Resistor (R):**

$R = \frac{\rho l}{A} \Omega$

**Ohms Law:**

$I = \sigma E$

$V = IR$

$P = VI = I^2 R = \frac{V^2}{R}$

**Inductor:**

$v = L \frac{di}{dt} = N \frac{d\phi}{dt}$

$E = \frac{1}{2} Li^2$

**Capacitor (C):**

$q = Cv$

$i = C \frac{dv}{dt}$

$E = \frac{1}{2} Cv^2$

**Unilateral Network:** Network properties or characteristics change with the direction.
Example: Diode

**Bilateral Network:** Network properties or characteristics are same in either direction.
Example: Transmission line.

**Network simplification techniques**

**Resistors in Series:**

$$R_{EQ} = R_1 + R_2 + \cdots + R_n$$

**Resistors in Parallel:**

$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$
**Current Division:**

\[ I_1 = \frac{IR_2}{R_1 + R_2}; \quad I_2 = \frac{IR_1}{R_1 + R_2} \]

**Voltage Division:**

\[ V_1 = \frac{VR_1}{R_1 + R_2}; \quad V_2 = \frac{VR_2}{R_1 + R_2} \]

**Star - Delta:**

\[ R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \]
\[ R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \]
\[ R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \]
\[ R_1 = \frac{R_aR_b}{R_a + R_b + R_c} \]
\[ R_2 = \frac{R_bR_c}{R_a + R_b + R_c} \]
\[ R_3 = \frac{R_aR_c}{R_a + R_b + R_c} \]

For Capacitor just Reciprocate the terms,

\[ \frac{1}{C_1} = \frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c} \]

Resistors of equal values are increased by 3 times in \( Y - \Delta \).

Capacitors of equal values are decreased by 3 times in \( Y - \Delta \).

**Kirchhoff’s Current Law:**

Algebraic sum of currents meeting at a point is equal to 0.

**Kirchhoff’s Voltage Law:**

Algebraic sum of voltages in closed loop is equal to 0.

**Mesh Analysis:**

\[ e = b - (N - 1) \]

1. Considering the loops and find the currents according to it.
2. If ideal source of voltage is present in between the circuit, then consider the circuit after the ideal source present in it.
3. If there is any current source that is common for meshes → Use Super Mesh Analysis.

Mesh → KVL + Ohm’s Law
Super Mesh → KVL + KCL + Ohm’s Law

**Nodal Analysis:**

\[ e = N - 1 \]
1. When ideal voltage source is connected between two Non-Reference Nodes
   → Use Super Nodal Analysis.
   Nodal → KCL + Ohm’s Law
   Super Nodal → KCL + KVL + Ohm’s Law

**Circuit Theorems**

- **Superposition Theorem:**
  - In any linear circuit containing multiple independent sources, the current or voltage at any point in the network maybe calculated as algebraic sum of the individual contributions of each source acting alone.
  - When determining the contribution due to a particular independent source, voltage source are made zero (Short circuit) and current sources are made zero (Open Circuit). If any dependent source is there keep it as undisturbed.

  **Note:** Superposition principle applies only to the current and voltage in a linear circuit but it cannot be used to determine power because power is a non-linear function.

- **Thevenin’s Theorem:**
  In any linear, bidirectional circuit having more than one element, then it can be replaced by single equivalent circuit consisting of equivalent voltage source \( V_{th} \) in series with equivalent resistance \( R_{th} \).

**Procedure for finding \( R_{th} \):**

Three different types of circuits may be encountered in determining the resistance.

- **Case (i):** If the circuit contains only independent source and resistors, deactivate the sources and find \( R_{th} \) by circuit reduction technique.

  Voltage Source Short circuit Current Source Open circuit.

- **Case (ii):** If the circuit contains resistors, dependent and independent sources, Step1: Find \( V_{oc} \) and \( I_{sc} \) with the sources activated.

- **Step 2:**
  \[ R_{th} = \frac{V_{oc}}{I_{sc}} \]

- **Case (iii):** If the circuit resisters and only dependent source, Then, \( V_{oc} = 0 \) (Since there is no energy source).

  **Step 1:** Connect 1V voltage source to output terminals and find the current drawn by circuit (I).

  **Step 2:**
  \[ R_{th} = \frac{1V}{I} \Omega \]

- **Maximum Power transfer Theorem:**

  **Case (i):** Both source and loads are resistive
The power that is delivered to the load is
\[ p = i^2 R_L = \left( \frac{V_t}{R_t + R_L} \right)^2 R_L \]
Maximum Power delivered to load when \( R_t = R_L \). Maximum power is given by
\[ P_{\text{max}} = \frac{V_t^2 R_L}{(2R_L)^2} = \frac{V_t^2}{4R_L} \]

Case (ii): When both source and loads are complex.
\[ Z_t = R_t + jX_t \]
\[ Z_L = R_L + jX_L \]
The average power delivered to the load,
\[ P = \frac{1}{2} |I|^2 R_L. \]
For maximum average power transfer,
\[ X_L = -X_t \text{ and } R_L = R_t \text{ i.e., } Z_L = Z_t. \]

Case (iii): When load is purely real and source is complex (i.e., \( X_L = 0 \)).
For maximum power transfer
\[ R_L = \sqrt{R_t^2 + X_t^2} = |Z_t| \]

Millman’s Theorem:
\[ V' = \frac{V_1 G_1 + V_2 G_2 + \ldots + V_n G_n}{G_1 + G_2 + \ldots + G_n} \]
\[ I' = \frac{I_1 R_1 + I_2 R_2 + \ldots + I_n R_n}{R_1 + R_2 + \ldots + R_n} \]
\[ V_{oc} = I_{sh} R_{th} \]

Tellegan’s Theorem:
Algebraic sum of the powers in any circuit at any instant is zero.
\[ \sum_{k=1}^{n} V_k i_k = 0 \]
When current is entering at
positive terminal \( \rightarrow \) Power Absorbing
Negative terminal \( \rightarrow \) Power Delivering
Absorbing power = Delivering power

Duality Theorem:
\[ R \leftrightarrow G \]
\[ L \leftrightarrow C \]
\[ V \leftrightarrow I \]
\[ KVL \leftrightarrow KCL \]
Series \( \leftrightarrow \) Parallel
Thevinin \( \leftrightarrow \) Norton
Soft Set \( \leftrightarrow \) Cut Set
O.C \( \leftrightarrow \) S.C
Loop \( \leftrightarrow \) Node
Foster – 1 Form \( \leftrightarrow \) Foster – 2 Form
\[ \int v \, dt \leftrightarrow \int i \, dt \]
Closed switch \( \leftrightarrow \) Open Switch
Resonance

A.C circuits made up of resistors, inductors and capacitors are said to be resonant circuits when the current drawn from the supply is in phase with the impressed sinusoidal voltage. Then,

1. The resultant reactance or susceptance is zero.
2. The circuit behaves as a resistive circuit.
3. The power factor is unity

Series Resonance:

\[
\begin{align*}
I &= \frac{E}{R + j(LC)^{-1}} = \frac{E}{R + jX} \\
\text{At Resonance, } X &= 0. \\
\omega_0L &= \frac{1}{\omega_0C} \\
\text{Resonant Frequency} &= \omega_0 = \frac{1}{\sqrt{LC}} \\
I_m &= \frac{V}{R}
\end{align*}
\]

Resonance can be achieved by

1. Varying frequency \( \omega \)
2. 2. Varying the inductance \( L \)
3. Varying the capacitance \( C \)

The current in the circuit is

\[ I = \frac{E}{R + j(LC)^{-1}} \]

Selectivity is the reciprocal of \( Q \).

Quality Factor:

\[ Q = \frac{2\pi \times \text{Maximum energy stored}}{\text{Total energy lost in a period}} \]

\[ Q = \frac{\omega_0L}{R} = \frac{1}{\omega_0CR} \]

At \( \omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow Q = \frac{1}{\sqrt{L/C}} \]

Bandwidth:

\[ \text{BW} = \omega_2 - \omega_1 = B. \] Half power frequencies \( \omega_1 \) and \( \omega_2 \)

\[ Q = \frac{\omega_0}{B} \Rightarrow B = \frac{\omega_0}{Q} \]

\[ B = \frac{R}{L}; \quad \omega_0 = \sqrt{\omega_1 \omega_2} \]

Parallel Resonance:

The dual of a series resonant circuit is often considered as a parallel resonant circuit.

\[ I_m = \frac{V}{R} \]

Resonance occurs at \( \omega_0 \), then the susceptance \( B \) is zero.
\[ \omega_0 L = \frac{1}{\omega_0 C} \]
\[ \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec} \]

**Variation of Voltage with frequency:**

\[ V = f \]
\[ \begin{array}{c}
\omega_1 \to 0 \\
\omega_2 \\
\end{array} \]

**Quality Factor:**

\[ Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Total energy lost in a period}} \]
\[ Q = \omega_0 RC = \frac{R}{\omega_0 L} \]

At \[ \omega_0 = \frac{1}{\sqrt{LC}} \]
\[ Q = \frac{R}{L} \sqrt{LC} = R \left[ \frac{C}{L} \right] \]

Bandwidth \( B = \omega_2 - \omega_1 = \frac{1}{RC} \)

---

**Two-Port Networks**

- **Z-parameters:**
  \[ V_1 = Z_{11} I_1 + Z_{12} I_2 \]
  \[ V_2 = Z_{21} I_1 + Z_{22} I_2 \]
  Dependent Variables: \( V_1, V_2 \)
  Independent Variables: \( I_1, I_2 \)

- **Y-parameters:**
  \[ I_1 = Y_{11} V_1 + Y_{12} V_2 \]
  \[ I_2 = Y_{21} V_1 + Y_{22} V_2 \]
  Dependent Variables: \( I_1, I_2 \)
  Independent Variables: \( V_1, V_2 \)

- **h-parameters:**
  \[ V_1 = h_{11} I_1 + h_{12} V_2 \]
  \[ I_2 = h_{21} I_1 + h_{22} V_2 \]
  Dependent Variables: \( V_1, I_2 \)
  Independent Variables: \( I_1, V_2 \)

- **g-parameters:**
  \[ I_1 = g_{11} V_1 + g_{12} I_2 \]
  \[ V_2 = g_{21} V_1 + g_{22} I_2 \]
  Dependent Variables: \( I_1, V_2 \)
  Independent Variables: \( V_1, I_2 \)

- **ABCD-parameters:**
  \[ V_1 = AV_2 - BL_2 \]
  \[ I_1 = CV_2 - DL_2 \]
  Dependent Variables: \( V_1, I_1 \)
  Independent Variables: \( V_2, I_2 \)
abcd-parameters:

\[ V_2 = aV_1 - bI_1 \]
\[ I_2 = cV_1 - dI_1 \]

Dependent Variables: \( V_2, I_2 \)
Independent Variables: \( V_1, I_1 \)

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<tr>
<td>( Y )</td>
<td>( Y_{11}=Y_{22} )</td>
<td>( Y_{12}=Y_{21} )</td>
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**Network Topology**

Network Topology: Is another method of solving electric circuits.

Network: A combination of two or more network elements is called a network.

Topology: Topology is a branch of geometry which is concerned with the properties of a geometrical figure.

Circuit: If the network has at least one closed path it is a circuit.

Branch: Representation of each element (component) of an electric network by a line segment is a branch.

Node: A point at which two or more elements are joined is a node.

Graph: It is a collection of branches and nodes in which each branch connects two nodes.

Graph of a Network: The diagram that gives network geometry and uses lines with dots at the ends to represent network element is usually called a graph of a given network.

Sub-graph: A sub-graph is a subset of branches and nodes of a graph.

Connected Graph: If there exists at least one path from each node to every other node

Path: A sequence of branches going from one node to other is called path.

Loop: Loop may be defined as a connected sub-graph of a graph

Planar and Non-planar Graphs: A planar graph is one where the branches do not cross each other while drawn on a plain sheet of paper. If they cross, they are non-planar.

Oriented Graph: The graph whose branches carry an orientation is called an oriented graph.

Tree: Tree of a connected graph is defined as any set of branches, which together connect all the nodes of the graph without forming any loops. The branches of a tree are called Twigs.

Co-tree: Remaining branches of a graph, which are not in the tree form a co-tree. The branches of a co-tree are called links or chords.

Properties of Tree:
1. It contains all the nodes of the graph.
2. It contains \((N - 1)\) branches. Where ‘\(N\)’ is total number of nodes in the given graph.
3. There are no closed paths.
4. Total number of tree branches, $n = N - 1$
5. Where $n_t =$ Total number of nodes
6. Total number of links,
   \[ l = b - (N - 1) \]
   Where $b =$ Total number of branches in the graph.
7. Total number of possible trees $N^{N-2}$
8. Total number of possible trees
   \[ = \text{det} \left( AA^T \right) \]
9. Total no. of node pair voltage
   \[ = \frac{N(N-1)}{2} \]
10. Total no. of edges $= \frac{N(N-1)}{2}$

**Degree of Node:** The number of branches attached to the node is degree of node.

**Tie-Set Matrix:**
1. It is a fundamental loops or f-loops or independent loops
2. Total no. of basic loops = Total no. of links
3. Loop direction is same as link current direction.
4. $[C] = [U; C_b]$
5. Total no. of possible Tie-Set matrix $= N^{N-2}$
6. Rank of Tie-Set matrix = Total No. of links
   \[ l = b - (N - 1) \]

**Cut-Set Matrix:**
1. It is a fundamental cut sets or f-cuts.
2. Total no. of basic cut sets = Total No. of tree branches
3. $[B] = [B_f; U]$  
4. Total no. of possible Cut-Set matrix $= N^{N-2}$
5. Rank of Cut-Set matrix = Total No. of tree branches
   \[ l = (N - 1) \]

**FILTERS:**

**Low Pass Filter:**
1. **First Order:**
   \[ T.F = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1 + Ts} = \frac{1}{1 + RC} \]
   T is Time constant $= RC$

2. **Second Order:**
High Pass Filter:

1. First Order:

\[ T.F = \frac{V_{out}(s)}{V_{in}(s)} = \frac{Ts}{1+Ts} = \frac{RCs}{1+RCs} \]

\( T \) is Time constant=RC

2. Second Order:

Band Pass Filter:

Band Elimination Filter:

Notch Filter is a type Band elimination Filter which eliminates only few frequencies.

Magnetic Coupled Circuits

Ideal Transformer:

\( N_1 \): Primary winding
$N_2$: Secondary winding

Turn Ratio: $N = N_2 / N_1$

\[
\frac{v_2}{v_1} = \frac{i_1}{i_2} = \frac{N_2}{N_1}
\]

**Dot Convention:**

\[
\begin{align*}
L_1 & \quad L_2 \\
V_1 & \quad V_2
\end{align*}
\]

\[
M_{12} = M_{21} = M
\]

\[
V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}
\]

\[
V_1 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}
\]

Relation between self and Mutual Inductance:

\[
M = k \sqrt{L_1 L_2}
\]

Coefficient of coupling:

\[
k = \sqrt{k_1 k_2} (0 < k < 1)
\]

For Ideal System $k = 1$.

**For Series Aiding:**

\[
L_{eq} = L_1 + L_2 + 2M
\]

**For Series Opposing:**

\[
L_{eq} = L_1 + L_2 - 2M
\]

**For Parallel Aiding:**

\[
L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}
\]

**For Parallel Opposing:**

\[
L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}
\]

**Transients**

Transients are available in network only when the network is having any energy storage elements.

When network having only resistive elements → No Transients.

$t = 0^-$: Indicates just before operating the switch.

$t = 0^+$: Indicates immediately after operating the switch.

$t = \infty$: Indicates steady state condition.

**Source free RL circuit:**

\[
i(t) = I_0 e^{-\frac{Rt}{L}}
\]

\[
V_R = I_0 e^{-\frac{Rt}{L}}
\]

\[
V_L = I_0 e^{-\frac{Rt}{L}}
\]

Current direction does not change. Voltage across inductor polarities are reversed.
RL circuit with source:

\[ i(t) = [i(0^+) - i(\infty)]e^{-\frac{R}{L}t} + i(\infty) \]

\[ i(t) = \frac{V}{R} \left[ 1 - e^{-\frac{R}{L}t} \right] \]

\[ V_R = V \left[ 1 - e^{-\frac{R}{L}t} \right] \]

\[ V_L = Ve^{-\frac{R}{L}t} \]

Source free RC circuit:

\[ v(t) = V_0 e^{-\frac{t}{RC}} \]

\[ I_R = \frac{V_0}{R} e^{-\frac{t}{RC}} \]

\[ I_C = -\frac{V_0}{R} e^{-\frac{t}{RC}} \]

Voltage across the capacitor polarities does not change. Current direction of capacitor is reversed.

RC circuit with source:

\[ i(t) = Ae^{-\frac{t}{RC}} \]

\[ i(t) = \frac{V}{R} e^{-\frac{t}{RC}} \]

\[ V_R = iR \quad , \quad V_R = Ve^{-\frac{t}{RC}} \]

\[ V_C = -Ve^{-\frac{t}{RC}} + V \]

\[ V_C(t) = [V_C(0^+) - V_C(\infty)]e^{-\frac{t}{RC}} + V_C(\infty) \]

**Time Constant:**

Charging or discharging action of energy state elements. Time Taken for response to rise 63.2% of maximum value.

\[ i(t) = \frac{V}{R} \left[ 1 - e^{-\frac{t}{T}} \right] \quad , \quad T = RC \quad , \quad T = \frac{L}{R} \]

**RLC series circuit with DC excitation:**

Over Damping

\[ \left( \frac{R}{2L} \right)^2 > \frac{1}{LC} \]

Critical Damping

\[ \left( \frac{R}{2L} \right)^2 = \frac{1}{LC} \]

Under Damping

\[ \left( \frac{R}{2L} \right)^2 < \frac{1}{LC} \]

Damping coefficient = \( R/2L \)

Time Constant = 1/Damping coefficient.

Parallel \( k = \frac{1}{2R} \sqrt{\frac{L}{C}} \)

Series \( \xi = \frac{R}{2} \sqrt{\frac{C}{L}} \)

For Over Damping \( \xi > 1 \)
For Under Damping \( \xi < 1 \)
For critical Damping \( \xi = 1 \)
For Un damping \( \xi = 0 \)

L\( \rightarrow \) Uncharged (i=0)
L\( \rightarrow \) Charged (I\( _L \)=I\(_S \))
C\( \rightarrow \) Charged (V\(_C \)=V)
C\( \rightarrow \) Uncharged (V\(_C \)=0)
Control Systems

Basics of Control Systems

Mechanical Translational system:
Input  → Force (F)
Output → Linear Displacement(x) or Linear Velocity (v)

\[ F = M \frac{d^2x}{dt^2} \quad \text{(Mass)} \]
\[ F = B \frac{dx}{dt} \quad \text{(Damper)} \]
\[ F = Kx \quad \text{(Spring)} \]

Mechanical Rotational system:
Input  → Torque (T)
Output → Angular Displacement(x) or Angular Velocity (v)

\[ F = J \frac{d^2\theta}{dt^2} \quad \text{(Mass)} \]
\[ F = B \frac{d\theta}{dt} \quad \text{(Damper)} \]
\[ F = K\theta \quad \text{(Spring)} \]

Analogous System:
Mechanical Translational system

\[ F = M \frac{d^2x}{dt^2} + B \frac{dx}{dt} + kx \]

Mechanical Rotational system

\[ F = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} + k\theta \]

Electrical System:
Series RLC

\[ V = L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} \]

Parallel RLC

\[ I = C \frac{d^2\theta}{dt^2} + \frac{1}{R} \frac{d\theta}{dt} + \frac{\theta}{L} \]

- Mass and spring are conservative elements due to their storage capacities

<table>
<thead>
<tr>
<th>Force</th>
<th>Torque</th>
<th>Voltage</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>J</td>
<td>L</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>R</td>
<td>( \frac{1}{R} )</td>
</tr>
<tr>
<td>K</td>
<td>K</td>
<td>( \frac{1}{C} )</td>
<td>( \frac{1}{L} )</td>
</tr>
<tr>
<td>V</td>
<td>w</td>
<td>i</td>
<td>V</td>
</tr>
<tr>
<td>x</td>
<td>( \theta )</td>
<td>q</td>
<td>( \theta )</td>
</tr>
</tbody>
</table>

Signal Flow Graphs

Input node → Outgoing Branches
Output node → Incoming Branches

- Self loops are not valid in Input Node side.
- Mason’s Gain formula

\[ T \cdot F = \frac{\sum P_k A_k}{\Delta} \]

Interacting System

If two or more processes are connected in series, then the overall transfer function is not the product of 2 individual transfer function.
**Non Interacting System**

If two or more processes are connected in series, then the overall transfer function is the product of 2 individual transfer functions.

---

**Time Domain Analysis**

Transient State → Nature of Response
Steady State → Estimation of Magnitude

**Standard Test Signals:**

- Sudden input → Step
- Velocity type input → Ramp
- Acceleration Type → Parabolic
- Sudden Shock → Impulse

Open Loop TF determines Steady state analysis and Type of system.

\[ T.F = \frac{k(1 + T_a s)}{s^p(1 + T_1 s)} \]

- Closed loop TF provides order of system and Transient state analysis.
- Highest power of Characteristic equation determines the order of the system.
  
  Order : \(1 + G(S)H(S) = 0\)

**Type :** \(G(S)H(S)\)

- Steady state analysis
  
  \[ e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s)H(s)} \]

with Disturbance:

\[ e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s)H(s)} \]

---

**Steady State Error for different inputs:**

**Step:**

\[ R(s) = \frac{A}{s} \]

\[ e_{ss} = \frac{A}{1 + k_p} \]

\[ k_p = \lim_{s \to 0} G(s)H(s) \]

**Ramp:**

\[ R(s) = \frac{A}{s^2} \]

\[ e_{ss} = \frac{A}{k_v} \]

\[ k_v = \lim_{s \to 0} s \cdot G(s)H(s) \]

**Parabolic:**

\[ R(s) = \frac{A}{s^3} \]

\[ e_{ss} = \frac{A}{k_a} \]

\[ k_a = \lim_{s \to 0} s^2 \cdot G(s)H(s) \]

\(k_p\) is position error constant
\(k_v\) is velocity error constant
\(k_a\) is acceleration error constant

- Steady State Error for different systems:

\[ G(S)H(S) = \frac{k(1 + T_a s)}{1 + T_1 s} \]

<table>
<thead>
<tr>
<th>Type</th>
<th>Type 0</th>
<th>Type 1</th>
<th>Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step</td>
<td>(\frac{A}{1 + k})</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ramp</td>
<td>(\infty)</td>
<td>(\frac{A}{k})</td>
<td>0</td>
</tr>
</tbody>
</table>
Error Series:
\[ e_{ss} = \lim_{t \to \infty} e(t) \]

Dynamic error constants:
\[ k_0 = \lim_{s \to \infty} F(s) \]
\[ k_1 = \lim_{s \to \infty} \frac{d}{ds} F(s) \]
\[ k_2 = \lim_{s \to \infty} \frac{d^2}{ds^2} F(s) \]
\[ F(s) = \frac{1}{1 + G(s)H(s)} \]

Transient state Analysis:
\[ X_0(s) = \frac{b_o}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_o} \]

Zero order:
\[ \frac{X_0(s)}{X_i(s)} = \frac{b_o}{a_o} = k \]

First order:
\[ \frac{X_0(s)}{X_i(s)} = \frac{k}{T_s + 1} \]
\[ x_o(t) = k(1 - e^{-\frac{t}{T_s}}) \]

'T' Time taken by response of system to reach 63% of the final value
\[ x_o(t) = k_o + (k_i - k_o)e^{-\frac{t}{T_s}} \]

Second order system:
\[ C(s) = \frac{\omega_n^2}{s^2 + 2\xi \omega_n s + \omega_n^2} \]
\[ R(S) = \frac{\omega_n^2}{s^2 + 2\xi \omega_n s + \omega_n^2} \]
\[ \xi = 0 \rightarrow \text{Undamped} \]

0 < \xi < 1 \rightarrow \text{Under damped}
\[ \xi = 1 \rightarrow \text{Critically damped} \]
\[ \xi > 1 \rightarrow \text{Over damped} \]

Optimum value of \( \xi = 0.3 \) to 0.7

Characteristics of Under damped system:
\[ \theta = \cos^{-1} \xi \]
\[ -\xi \omega_n \pm \omega_n \sqrt{1 - \xi^2} \]
\[ \alpha = \xi \omega_n \]
\[ T = \frac{1}{\alpha} \]
\[ \omega_d = \omega_n \sqrt{1 - \xi^2} \]
\[ (s + \xi \omega_n)^2 + \omega_n^2 \]

Actual Damping= \( \xi \omega_n \)
Critical Damping= \( \omega_n \)

Transient Analysis:

Delay time (\( T_d \)) = \( \frac{1 + 0.7\xi}{\omega_n} \)

Rise time (\( T_r \)) = \( \frac{\pi - \theta}{\omega_d} \),
\[ \theta = \tan^{-1} \sqrt{\frac{1 - \xi^2}{\xi}} \]

Peak time (\( T_p \)) = \( \frac{n\pi}{\omega_d} \)

Settling time (\( T_s \)) = \( \frac{1}{\xi \omega_n} \)

Maximum Peak Overshoot (MP):
\[ M.P = \frac{e^{-\frac{-\xi \pi}{\omega_n}}}{\sqrt{1 - \xi^2}} \]
\[ \% M.P = \frac{e^{-\frac{-\xi \pi}{\omega_n}}}{\sqrt{1 - \xi^2}} \times 100\% \]
\[ c(t) = 1 - \frac{e^{\xi \omega_n t}}{\sqrt{1 - \xi^2}} \sin(\omega_d t + \theta) \]

No. of damped oscillations,
Higher order system is done by approximating to second order system with respect to dominant poles. Two poles should lie in the dominant region. So, for any higher order system consider only second order system.

### Stability

**Routh-Hurwitz Stability Criterion:**
- The R-H criterion is a method for determining whether a linear system is stable or not.
- Characteristic Equation:
  \[
  C.E = 1 + G(s)H(s)
  
  C.E = a_nS^n + a_{n-1}S^{n-1} + a_{n-2}S^{n-2} 
  + \cdots + a_1S + a_0 = 0
  \]
- To determine whether this system is stable or not, check the following conditions:
- Two necessary but not sufficient conditions that all the roots have negative real parts are
  - (a) All the polynomial coefficients must have the same sign.
  - (b) All the polynomial coefficients must be nonzero.
- The necessary condition that all roots have negative real parts is that all the elements of the first column of the array have the same sign. The number of changes of sign equals the number of roots with positive real parts.
- **Special Case 1:** The first element of a row is zero, but some other elements in that row are nonzero. In this case, replace zero by \( \xi \), then complete the table. The results must be interpreted in the limit as \( \xi \to 0 \).
- **Special Case 2:** If all the elements of a particular row are zero.
  \[
  A.E \to \frac{d(A.E)}{ds}
  \]
  Replace zeros with coefficients of
  \[
  \frac{d(A.E)}{ds}
  \]

### Frequency Domain Analysis
- This is analysis by varying \( \omega \) from 0 to \( \infty \).
  \[
  |F(j\omega)| = \sqrt{(r.p)^2 + (i.p)^2}
  \]
  \[
  \angle F(j\omega) = \tan^{-1}\left(\frac{i.p}{r.p}\right)
  \]
  Resonant Frequency \( \omega_r \):
  \[
  \omega_r = \omega_n\sqrt{1 - 2\xi^2}
  \]
  \[
  \omega_d = \omega_n\sqrt{1 - \xi^2}
  \]
  Resonant Peak \( M_r \):
  \[
  M_r = \frac{1}{2\xi\sqrt{1 - \xi^2}}
  \]
  \[
  \xi > \frac{1}{\sqrt{2}}; M_r (not needed)
  \]
\[ \xi = \frac{1}{\sqrt{2}}, M_r = 1 \]
\[ \xi < \frac{1}{\sqrt{2}}, M_r > 1 \]

**Bandwidth:**
Indicates speed of response

Wider bandwidth (Faster response)

\[ B.W \propto \frac{1}{t_r} \]

Cut Off frequency:
\[ \omega_c = \omega_n \sqrt{1 - 2\xi^2 + \sqrt{4\xi^4 - 4\xi^2 + 2}} \]
\[ \cong B.W \]

- **Polar plot:** \( |F(j\omega)| \angle F(j\omega) Vs \omega \).
- **Bode plot:** \(|F(j\omega)| \angle F(j\omega) Vs \log \omega \)
  - \( F(s) = K \)
    \[ |F(j\omega)|_{dB} = 20 \log K \]
    slope = 0dB/dec
  - Poles and Zeros at origin
    \( (s)^{\pm n} \)
    \( +n \rightarrow \text{zeros} \)
    \( -n \rightarrow \text{poles} \)
    \[ |F(j\omega)|_{dB} = \pm 20n \log(\omega) \]
    slope = \( \pm 20n \log(\omega) \)dB/dec
  - First order T/F:
    \[ \frac{C(S)}{R(S)} = \frac{1}{1 + ST} \]
    Slope = \( \pm 20 \)dB/dec
  - Corner Frequency \( = \omega_{ef} = \frac{1}{t} \)
  - 2\( ^{nd} \) Order T/F:
    \[ \frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi \omega_n + \omega_n^2} \]
    slope = \(-40 \)dB/dec

Corner Frequency \( = \omega_{ef} = \omega_n \)

Error at Corner Frequency
\[ = -20 \log 2\xi \]

- Gain Cross Over Frequency \( (\omega_{gc}) \):
  \[ |G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 1 \]
- Phase Cross Over Frequency \( (\omega_{pc}) \):
  \[ \angle G(j\omega)H(j\omega) = -180 \, @ \, \omega = \omega_{pc} \]
- Gain Margin
  \[ |G(j\omega)H(j\omega)| = X; \, GM = \frac{1}{X} \]
  \[ \therefore G.M(dB) = 20 \log \frac{1}{X} \]
- Phase Margin
  \[ \angle G(j\omega)H(j\omega) = \phi \]
  \[ \therefore P.M = 180 + \phi \]
  - If G.M and P.M are +ve \( i.e. \omega_{gc} < \omega_{pc} \)
    then system is stable
  - If G.M and P.M are -ve \( i.e. \omega_{gc} > \omega_{pc} \)
    then system is unstable
  - If G.M and P.M are 0 \( i.e. \omega_{gc} = \omega_{pc} \)
    then system is marginally stable.

**Nyquist Plot:**
\[ N = P - Z \]
\[ N = \text{No.of Encirclements.} \]
\[ P = \text{No.of Open loop poles in RHS of s-plane} \]
\[ Z = \text{No.of Closed Loop Poles in RHS of S-Plane} \]

**Stability Criteria:**
If \( N=P \) closed loop system is stable.
Addition of a Pole in terms of compensators will be lag compensator.
Improving the steady state characteristics and eliminate steady state errors.

- **Lead-Lag (or) Lag-Lead:**

\[
T.F = \frac{\alpha(1 + T_1S)(1 + T_2S)}{(1 + \alpha T_1S)(1 + \beta T_2S)}; T_2 > T_1
\]

### Controllers

- **P controller**
  → ON-OFF controller

  → Offset error, \( e_{ss} = \frac{A}{1 + kp} \)

- **Integral Mode** → Reset time, \( e_{ss} = 0 \)
  → Response is very slow.

- **Derivative Mode** → Rate time
  → Anticipatory Action
  → Results with more instability
  → Cannot respond to sudden error.

- **PI controller**
  It is a lag Compensator.

\[
p = k_p e + \frac{k_p}{T_i} \int e \, dt
\]

\[
k_p = \frac{R_2}{R_1}, \quad T_i = R_2 C
\]

Improvement in steady state Response.
Integral eliminates the Offset

\[
P(s) = k_p \left[ 1 + \frac{1}{T_i s} \right] E(s)
\]

Acts as Low Pass filter
Rise time (\( T_r \)) increases, Bandwidth and Stability decreases.

### Compensators

- **Lead:**

\[
T.F = \frac{\alpha(1 + TS)}{1 + \alpha TS}
\]

\[
T = R_1 C; \quad \alpha = \frac{R_2}{R_1 + R_2}
\]

Addition of a Zero in terms of compensators will be lead compensator.
Improving the speed of response or the transient state of system

- **Lag:**

\[
T.F = \frac{1 + TS}{(1 + \beta TS)}
\]

\[
T = R_2 C; \quad \beta = \frac{R_1 + R_2}{R_2}
\]
• **PD controller**
  It is a lag Compensator.
  \[ p = k_p e + k_p T_d \frac{de}{dt} \]
  \[ k_p = \frac{R_2}{R_1}, \quad T_d = R_1 C \]
  \[ P(s) = k_p [1 + T_d s] E(s) \]
  Improving Transient Response (or) speed.
  High Pass Filter.
  Rise time Decreases, Bandwith and Stability increases.

• **PID Controller**
  It is a Lead-lag Compensator.
  \[ p = k_p e + \frac{k_p}{T_i} \int e \, dt + k_p T_d \frac{de}{dt} \]
  \[ k_p = \frac{R_2}{R_1}, \quad T_d = R_1 C_1, \quad T_i = R_2 C_2 \]
  \[ P(s) = k_p \left[ 1 + \frac{1}{T_i s} + T_d s \right] E(s) \]
  Both Responses are improved.
  It is a Band stop or Band reject filter.
  Eliminates steady state error.

---

**State Space Analysis**

State Equation: \( \dot{x}(t) = Ax(t) + Bu(t) \)

O/p Equation: \( y(t) = Cx(t) + Du(t) \)

---

• **Transfer Function:**
  \[ \frac{Y(s)}{U(s)} = C [SI - A]^{-1} B + D \]

• **Controllability:**
  \[ Q_c = [B \ AB A^2 B ... A^{n-1} B] \]
  \[ |Q_c| \neq 0 \rightarrow \text{For controllable Systems} \]

• **Observability:**
  \[ Q_o = [C^T A^T C^T (A^T)^2 C^T ... (A^T)^{n-1} C^T] \]
  \[ |Q_o| \neq 0 \rightarrow \text{For Observable Systems} \]

• **Zero Input (only initial conditions) Response:**
  \[ X(s) = [SI - A]^{-1} X(0) \]
  \[ x(t) = e^{At} X(0) \]

• **State Transition Matrix:**
  \( e^{At} = \phi(t) = L^{-1} \{ [SI - A]^{-1} \} \)

• **Properties of State Transition Matrix:**
  \( \phi(0) = I \)
  \( \phi^{-1}(t) = \phi(-t) \)
  \( [\phi(t)]^k = \phi(kt) \)
  \( \phi(t_2 - t_1)\phi(t_1 - t_0) = \phi(t_2 - t_0) \)

• **Forced Response (due to input):**
  \[ x(t) = L^{-1} \{ [SI - A]^{-1} X(0) \}
  \[ + L^{-1} \{ [SI - A]^{-1} Bu(s) \} \]
Electronics Devices

Introduction to Semiconductor Physics

Atomic Model:
The maximum no. of electrons present in each orbit is represented by \(2n^2\)
where \(n\) = no. of orbits.

\[E_G(\text{Ge}) < E_G(\text{Si})\]

The valance electron in Ge is far away from the nucleus than Si.

The energy gap at any temperature

\[E_G(T) = E_{GO} - \beta T\]

Where, \(E_{GO}\) = Energy gap at 0°C
\(\beta = A\) constant
\(3.6 \times 10^{-4} - \text{Si}\)
\(2.23 \times 10^{-4} - \text{Ge}\)

<table>
<thead>
<tr>
<th></th>
<th>(E_{G0})</th>
<th>(E_G(T))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>1.21 eV</td>
<td>1.11 eV</td>
</tr>
<tr>
<td>Ge</td>
<td>0.78 eV</td>
<td>0.71 eV</td>
</tr>
<tr>
<td>GaAs</td>
<td>-</td>
<td>1.43 eV</td>
</tr>
</tbody>
</table>

\[v = -\mu E\]

Here \(-\) indicates the electron direction is opposite to the electric field direction.

\[\mu_n = \frac{qT_n}{m_n}\]

\(E = \text{Electric Field}\)

\[E = \frac{V}{d}\]

Drift Velocity Vs Electric Field

Mobility Vs Temperature

\[\mu \propto T^{-m}\]

At room temperature, \(\mu \propto T^{-m}\)

\(m = 2.5(2.7)\) for electrons (holes) – Si

\(= 1.66(2.33)\) for electrons (holes) – Ge
Mobility Vs Electric Field

\[ \mu = \text{Constant, } E < 10^3 \text{ V/cm} \]
\[ -E^{-1/2} < 10^3 < E < 10^4 \text{ V/cm} \]
\[ \frac{1}{E} < E > 10^4 \text{ V/cm} \]

Current density

\[ J = \frac{1}{A} \text{ A/m}^2 \text{ (or) A/cm}^2 \]

In metals, \( J = nq\mu E \)

\[ J = \sigma E \]

where, \( \sigma = \text{Conductivity} \)
\[ \sigma = nq\mu \]
\[ J = \rho \nu \]

Where, \( \rho = \text{charge density in c/m}^3 \)
\[ \rho = nq \]

The Fermi Dirac Function

\[ f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \]

Case (i): Let \( T = 0^\circ K \) and \( E < E_F \) then

\[ f(E) = \frac{1}{1 + e^{-\infty}} = \frac{1}{1 + 0} = 1 \]

i.e., All energy levels below \( E_F \) are filled with electrons at \( 0^\circ K \)

Case (ii): Let \( T = 0^\circ K \) and \( E > E_F \) then

\[ f(E) = \frac{1}{1 + e^{\infty}} = \frac{1}{1 + 0} = 1 \]

The Fermi energy is the energy level at which the probability of finding an electron is 0.5.

Case (iii): If \( T > 0 \) and \( E = E_F \) then

\[ f(E) = \frac{1}{1 + e^0} = 0.5 \]

i.e., At a non-zero temperature, Fermi energy is the energy level at which the probability of finding an electron is 0.5.

For various temperatures
**Intrinsic Semiconductor:**

- In Intrinsic semiconductor the free electron concentration in conduction band is equal to hole concentration in valance band.
  
  \[ n = p = n_i \]

- The electron concentration in conduction band is
  
  \[ n = N_e e^{-(E_C - E_F)/kT} \]

- The hole concentration in valance band is
  
  \[ p = N_v e^{-(E_F - E_V)/kT} \]

Where, \( N_C \) = Total effective density of states in Conduction Band.

\[ N_C = 2 \left( \frac{2\pi m^*_n \times kT}{h^2} \right)^{3/2} \]

\( m_n^* \) : The Effective mass of a free electron

\( m_n \) : The true mass of electron when there is no electric field.

\( N_V \) : Total effective density of states in Valence Band.

\[ N_V = 2 \left( \frac{2\pi m^*_p \times kT}{h^2} \right)^{3/2} \]

\( m_p^* \) : The Effective mass of a hole.

**Mass Action Law:**

\[ n \times p = n_i^2 \]

Intrinsic concentration,

\[ n_i^2 = A_0 T^3 e^{-E_G/kT} \]

- As \( n_i(\text{Ge}) > n_i(\text{Si}) \), the silicon has more breakdown voltage than Germanium.
  
  i.e., \( V_{BR(\text{Si})} > V_{BR(\text{Ge})} \)

- Electron and hole direction are always opposite in direction.

- The condition for charge neutrality is
  
  \[ N_D + p = N_A + n \]

- The Fermi level lies at the centre of forbidden band in Intrinsic Semiconductor.

\[ E_F = \frac{E_C + E_V}{2} \]

- Intrinsic semiconductor is a NTC device.

\[ T \uparrow \Rightarrow n_i^2 \uparrow \Rightarrow \sigma_i \uparrow \Rightarrow p_i \downarrow \Rightarrow R \downarrow \]

\[ J_i = \sigma_i E \]

Where \( \sigma_i = n_i q (\mu_n + \mu_p) \)
n-type Semiconductor:

- The pentavalent impurities are P, As, Sb.
  - Majority concentration is directly proportional to doping concentration.

\[
n_n = N_D \quad N_D \gg n_i
\]

\[
= \frac{N_D}{2} + \sqrt{\frac{N_D^2 + 4n_i^2}{2}} \quad N_D \ll n_i
\]

- Minority concentration is inversely proportional to doping concentration.

\[
p_n = \frac{n_i^2}{N_D}
\]

- The current density is

\[
J_n = \sigma_n E
\]

Where \( \sigma_n = N_D \mu_n \)

- The Fermi level \( n \)-type semiconductor lies just below the conduction band.

\[
E_{Fi} = E_C - kT \ln \left( \frac{N_C}{N_D} \right)
\]

\[
E_{Fi} = E_F + kT \ln \left( \frac{N_D}{n_i} \right)
\]

p-type Semiconductor:

- The trivalent impurities are B, Ga, In, Al.
  - Majority concentration is directly proportional to doping concentration.

\[
p_p = N_A \quad N_A \gg n_i
\]

\[
= \frac{N_A}{2} + \sqrt{\frac{N_A^2 + 4n_i^2}{2}} \quad N_A \ll n_i
\]

- Minority concentration is inversely proportional to doping concentration.

\[
n_p = \frac{n_i^2}{N_A}
\]

- The current density is

\[
J_p = \sigma_p E
\]

Where \( \sigma_p = N_A \mu_p \)

- The Fermi level p-type semiconductor lies just above the valence band.

\[
E_{Fp} = E_V + kT \ln \left( \frac{N_V}{N_A} \right)
\]

\[
E_{Fp} = E_F - kT \ln \left( \frac{N_A}{n_i} \right)
\]
i.e., As doping increases, the Fermi level moves away from the intrinsic Fermi level and intrinsic semiconductor becomes extrinsic semiconductor.

As temperature increases, the Fermi level moves towards the Fermi level and extrinsic semiconductor becomes intrinsic semiconductor.

- Metals and Extrinsic semiconductor are PTC devices.
- As $\mu_n > \mu_p$, $R_p > R_n$
- For a compensated semiconductor
  - Majority concentration
    
    $$ = N_D - N_A \text{ if } N_D > N_A $$
    $$ = N_A - N_D \text{ if } N_A > N_D $$
  - Minority concentration,
    
    $$ = \frac{n_i^2}{n_i} $$

**Drift and Diffusion Currents:**
- The current due to the external voltage (or potential gradient) is called drift current.
- Drift current is present both in metals and semiconductors.
  
  $$ J_{\text{drift}} = J_{\text{n,drift}} + J_{\text{p,drift}} $$
  
  $$ J_{\text{drift}} = nq\mu_n E + pq\mu_p E $$
- The current due to the concentration gradient and without application of voltage is called diffusion current.

**Diffusion Current:**
- Diffusion current is present only in non-uniformly doped semiconductor.
- The diffusion current in a uniformly doped semiconductor is zero.
- The electron diffusion current is
  
  $$ J_{\text{n, diffusion}} = q D_n \frac{dn}{dx} $$
- The hole diffusion current is
  
  $$ J_{\text{p, diffusion}} = -q D_p \frac{dp}{dx} $$

Where, $D_p$ (or) $D_n$ - Diffusivity (or) Diffusion constant (or) Diffusion co-efficient of a hole (or) electron (m$^2$/sec (or) cm$^2$/sec)

**Einstein Relationship:**

$$ \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T $$

In other words,

$$ \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = kT = \frac{\bar{KT}}{q} = V_T = \frac{T}{11600} $$

Where, $T$ = Temperature in °K

**Note:**
- $V_T$ at $T = 0^\circ K$ is 23V
- $V_T$ at $T = 300^\circ K$ is 26mV
- $V_T$ is not a strong function of temperature.
- The units of $\frac{q}{kT}$ is V$^{-1}$
- $\mu \approx 39D$
Hall Effect:
- The Hall voltage \( V_H = \frac{BI}{\rho W} \)
- Hall co-efficient \( R_H = \frac{1}{\rho} \)
\[ \therefore R_H = \frac{V_H W}{BI} \]
- Units of Hall co-efficient is \( m^3/C \)
- Mobility, \( \mu = \sigma R_H \)
- The Hall voltage is micro volts in Extrinsic semiconductors and milli volts in Intrinsic semiconductors.

Applications of Hall Effect:
- Hall field meter
- Hall multiplier
The Hall voltage is zero for insulator.

Carrier generation and Recombination:
\[ G_n = \frac{\text{no.of excess electrons generated}}{\text{mean life time}(\tau_n)} \]

Similarly, \( G_p = \frac{\Delta p}{\tau_p} \)

At equilibrium, the generation rate is equal to recombination rate
i.e., \( G_p = G_n \)
\[ \therefore \frac{\Delta p}{\tau_p} = \frac{\Delta n}{\tau_n} \]

\[ R_n = \frac{\text{no.of electrons recombined}}{\text{mean life time}(\tau_n)} \]

Similarly, \( R_p = \frac{\Delta P}{\tau_p} \)
i.e., \( R_p = R_n \)
\[ \therefore \frac{\Delta P}{\tau_p} = \frac{\Delta n}{\tau_n} \]

According to law of conservation of charge, the recombination rate of electrons is equal to generation rate of electrons.
\[ \therefore G_n = R_n \]
Similarly, \( G_p = R_p \)

### p-n Junction Diode Characteristics

- Depletion region contains only ions.
- Depletion region width,
\[ W = \frac{2\varepsilon}{q} V_{bi} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \]

Where, \( \varepsilon = \) permittivity of material (or)
Dielectric constant.
\[ \varepsilon = \varepsilon_0 \varepsilon_r \]
\( \varepsilon_0 = \) Permittivity of free space
\( \varepsilon_r = \) Relative permittivity of given material
\[ V_{bi} = \text{in-built potential (or) contact potential (} V_0 \text{)} \]

\[ W \propto \sqrt{V_{bi}} \]

\[ \propto \frac{1}{\sqrt{\text{Doping concentration}}} \]

- \( W = W_p + W_n \)
- \( W_p = W_n \)
- \( W \approx W_n \)  \( W_p \ll W_n \)

The depletion layer width for a \( p^+n \) diode is

\[ W = W_n = \sqrt{\frac{2e}{qN_D}} V_{bi} \]

\[ W \equiv W_p \]

\[ W_n \ll W_p \]

Similarly for a \( pn^+ \) diode

\[ W = W_p = \sqrt{\frac{2e}{qN_A}} V_{bi} \]

- \( N_D x_n = N_A x_p \)
- \( |E_{max}| = \frac{2V_{bi}}{W} \)
- \( \text{In-built potential, } V_{bi} \)

\[ V_{bi} = V_T \ln \left[ \frac{N_A N_D}{n_i^2} \right] \]

- As \( E_{GO(Ge)} < E_{GO(Si)} \), \( n_{i(Si)} < n_{i(Ge)} \)

Hence, as \( n_{i(Si)} < n_{i(Ge)} \), \( V_{bi(Si)} > V_{bi(Ge)} \)

\[ V_{bi} = 0.3V \rightarrow Ge \]

\[ = 0.7V \rightarrow Si \]

- As temperature increases, \( n_i \) value increases therefore \( V_{bi} \) value decreases.

- The depletion layer width for a \( p-n \) diode is \( 0.5\mu m \) to \( 1\mu m \)

- Electric field is maximum at the centre of \( p-n \) junction.

- In \( p-n \) junction diode with Forward bias the depletion layer width is
Electric Field is \[ |E_{\text{max}}| = \frac{2(V_{\text{bi}} - V_{FB})}{W} \]

In p-n junction diode with Reverse bias the depletion layer width is \[ W = \frac{2e(V_{\text{bi}} + V_{RB})}{q(N_A + N_D)} \]

Electric Field is \[ |E_{\text{max}}| = \frac{2(V_{\text{bi}} + V_{RB})}{W} \]

If total width is given then,

\[ W_p = \frac{W N_D}{N_A + N_D} \]
\[ W_n = \frac{W N_A}{N_A + N_D} \]

The diode current equation is,

\[ I = I_0 \left( e^{V/\eta V_T} - 1 \right) \]

Where, \( I_0 = \) Reverse saturation current

\[ I_0 = \frac{A q D_p N_D}{L_p N_D} + \frac{A q D_n N_A}{L_n N_A} \eta_i^2 \]

\( L_p, L_n = \) Diffusion length of holes and electrons.

\[ L_p = \sqrt{D_p \tau_p} \]
\[ L_n = \sqrt{D_n \tau_n} \]

\( \eta = \) Ideality factor

\( = 1 \) – Ge
\( = 2 \) – Si (low currents)
\( = 1 \) – Si (large currents)

The diode voltage equation is,

\[ V = \eta V_T \ln \left( \frac{1}{I_0} + 1 \right) \]

\[ V = 2.3 \eta V_T \log \left( \frac{1}{I_0} + 1 \right) \]

The change in the diodes voltage with respect to temperature is

\[ \frac{dV}{dT} = -2.5 \text{mV/°C} \]

Note: \( \frac{dV}{dT} = -2.1 \text{mV/°C} \) - Ge

\( = -2.3 \text{mV/°C} \) - Si

The change in the diodes reverse saturation current with respect to temperature is

\[ I_{02} = I_{01} \times 2^{(T_2-T_1)/10} \]

Note: \( = 7\% \) for 1°C (In general)

\( = 8\% \) for Si

\( = 11\% \) for Ge

The breakdown voltage of a \( p^+ - n \) diode

\[ V_{BR} = \frac{eE_{\text{crit}}^2}{2q N_D} \]

Diode dynamic resistance,

\[ r = \frac{\eta V_T}{1} \]

Depletion capacitance,

\[ C = \frac{\varepsilon A}{W} \]

Diffusion capacitance,
In general, Depletion capacitance is always greater than diffusion capacitance.

**Diode Switching Times:**

$$C_D = \frac{\tau I}{\eta V_I}$$

$$t_{rr} = t_s + t_t$$

$t_{rr}$ = reverse recovery time: It is the time taken by a diode from ON state to OFF state.

$t_{fr}$ : Forward recovery time: It is the time taken by a diode from OFF state to ON state.

In general, $t_{rr} > t_{fr}$

Where, $t_s$ – Storage time

$t_t$ – Transition time

Normally, $t_s < t_t$
### Special Purpose Diodes

<table>
<thead>
<tr>
<th>S.No</th>
<th>Name of the diode</th>
<th>Symbol</th>
<th>Biasing mode &amp; Cut-in Voltage</th>
<th>Basic Material</th>
<th>Basic Principle</th>
<th>Applications</th>
<th>V-I Char.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Varactor diode</td>
<td>K</td>
<td>Reverse bias</td>
<td>GaAs (or) Ge</td>
<td>High frequency application, FM reactance modulator</td>
<td></td>
<td><img src="image" alt="V-I Char." /></td>
</tr>
<tr>
<td>2</td>
<td>Zener diode</td>
<td>K</td>
<td>Reverse bias</td>
<td>Si</td>
<td>Voltage regulator</td>
<td></td>
<td><img src="image" alt="V-I Char." /></td>
</tr>
<tr>
<td>3</td>
<td>LED</td>
<td>A</td>
<td>Forward bias</td>
<td>GaAs – Infra red Gap Visible GaAsp light</td>
<td>Electroluminescence</td>
<td>Remote control units opto couplers</td>
<td><img src="image" alt="V-I Char." /></td>
</tr>
<tr>
<td>4</td>
<td>Solar Cell</td>
<td>light Solar cell 0.6V</td>
<td>No bias, preferably forward bias $V_j = 0$</td>
<td>Si</td>
<td>Photo voltaic effect</td>
<td>Satellites Automatic traffic signals lightening</td>
<td><img src="image" alt="V-I Char." /></td>
</tr>
<tr>
<td>5</td>
<td>Photo diode</td>
<td>K</td>
<td>Reverse bias</td>
<td>Si Ge</td>
<td>Photo conductive effect</td>
<td>Opto couplers</td>
<td><img src="image" alt="V-I Char." /></td>
</tr>
<tr>
<td>6</td>
<td>Schottky diode</td>
<td>K</td>
<td>Forward bias</td>
<td>Si</td>
<td>High frequency switching applications</td>
<td></td>
<td><img src="image" alt="V-I Char." /></td>
</tr>
<tr>
<td>7</td>
<td>Tunnel diode</td>
<td>K</td>
<td>Forward bias</td>
<td>GaAs (or) Ge</td>
<td>Tunneling effect</td>
<td>High Frequency Switching applications</td>
<td><img src="image" alt="V-I Char." /></td>
</tr>
</tbody>
</table>

#### LED:
- The relationship between energy of a photon wavelength $E_G (eV) = \frac{1.24}{\lambda (\mu m)}$
- $E = h\nu$
- Where $\nu$= frequency

#### Varactor diode:
- In varactor diode $C_j \propto V_j^{-n}$
- $n = \frac{1}{2}$ step graded junction.
- $n = \frac{1}{3}$ linear graded junction.
Solar cell:
- The photo current density \( J_L \) is
  \[ J_L = J_s e^{V_{oc}/V_T} \]
  Where \( J_s \) = minority current density
- The open circuit voltage,
  \[ V_{oc} = V_T \ln \left[ \frac{J_L}{J_s} \right] \]
- Fill Factor = \( \frac{\text{Max imum power rating} (P_{\text{max}})}{\text{Theoretical power} (P_T)} \)
  Where \( P_T = J_{sc} \times V_{oc} \)
  \( J_{sc} \) = short circuit current
  \( V_{oc} \) = open circuit voltage
- Conversion Efficiency
  \[ \eta_{\text{max}} = \frac{\text{Max imum power rating} (P_{\text{max}})}{\text{Input power} (P_{\text{in}})} \times 100 \]

Photo diode:
- In photo diode, we use convex lens to receive the light from all directions and to illuminate at the junction.
  \[ \text{Responsivity} (R) = \frac{\text{photo current} (I_p)}{\text{Incident power} (P_0)} \]

Tunnel diode:
- Typical tunnel diode parameters.
  - The negative resistance region lies in between \( V_P \leq V_D \leq V_V \)

**BJT Characteristics**

There are two types of transistors.
1. n-p-n transistor
2. p-n-p transistor

- Emitter is a heavily doped region.
- Base is a lightly doped region and it has minimum width.
- Collector is a medium doped region and it has maximum width (or) area.
  \[ \therefore N_B < N_C < N_E \]
  \[ W_B < W_E < W_C \]
- Here the arrow mark indicates the conventional current direction.
Various currents and voltages in a transistor:

\[ I_E = I_B + I_C \]
\[ V_{CE} = V_{CB} + V_{BE} \]

\( I_E \) and \( I_C \) are always opposite in direction for a given transistor.

For a BJT operated in active region.

\( W_{J_E} < W_{J_C} \)

Types of operating modes in a BJT:

<table>
<thead>
<tr>
<th>S.No</th>
<th>( J_E )</th>
<th>( J_C )</th>
<th>Type of Region</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>F.B</td>
<td>F.B</td>
<td>Saturation region</td>
<td>ON switch</td>
</tr>
<tr>
<td>2.</td>
<td>R.B</td>
<td>R.B</td>
<td>Cut off region</td>
<td>OFF switch</td>
</tr>
<tr>
<td>3.</td>
<td>F.B</td>
<td>R.B</td>
<td>Active region</td>
<td>Amplifier</td>
</tr>
<tr>
<td>4.</td>
<td>R.B</td>
<td>F.B</td>
<td>Inverse Active region</td>
<td>–</td>
</tr>
</tbody>
</table>

- The maximum power dissipation occurs in active region.

- Transistor is an electronic device which transfers the input signal from low resistance path to high resistance path.
  
i.e., transfer + resistor = Transistor

- We can analyze the given transistor may be equivalent to two diodes connected in back to back direction.

\[ I_C = \beta I_B + (1 + \beta) I_{CO} \]

- The various leakage currents in a transistor are

\( I_{CEO} = (1 + \beta) I_{CO} \)

\( I_{CBO} = I_{CO} + I_{surface leakage} \)

\( I_{CO} < I_{CBO} < I_{CEO} \)

The relation between \( \alpha, \beta, \gamma \) is

\[ \alpha = \frac{I_C}{I_E} \quad \beta = \frac{I_C}{I_B} \quad \gamma = \frac{I_E}{I_B} \]
By increasing $|V_{CB}|$, there is an increment in collector current of a BJT. This is called early effect.

It has three consequences:

1. $\alpha$ increases with a small value. Hence the CB configuration output characteristics are more flatter than CE (or) CC configuration.

2. $\beta$ increases with a large value. Hence the output characteristics of CE configuration has increasing slope.

3. Punch through effect may occur for a large value of $V_{CB}$.

**Def:** For a large value of $V_{CB}$, there may be short circuit of both collector and emitter regions which leads to damage of the transistor permanently.

$\beta$ or dc analysis is greater than $\beta$ for ac analysis.

\[ \beta_{dc} > \beta_{ac} \quad \text{or} \quad h_{FE} > h_{fe} \]

$\alpha$ can also be expressed as,

\[ \alpha = \beta^* \times \gamma \]

Where, $\beta^*$ = Base transportation factor

$\gamma$ = Emitter injection efficiency

Emitter junction breakdown is due to Zener effect while collector junction breakdown is due to avalanche effect.
FET refers Field Effect Transistor. It has three terminals similar to BJT:

- Source - Emitter
- Gate - Base
- Drain - Collector

The arrow mark indicates the conventional current direction when input junction is forward biased. But we never consider the input junction as Forward bias and it is always reverse bias.

n-channel JFET:

![n-channel JFET](image)

In JFET, input junction and output junction are reverse bias only.

Salient Features of FET:

- It is a symmetrical device hence occupy less space in IC fabrication.
- It is a voltage control device. In other words, voltage control current source.
- It is a PTC device.
- It is a zero offset device.
- It is a unipolar device.
- It has high input impedance than BJT.
- BJT is faster than FET devices but its power consumption is large compare to FET devices.

The drain current of a FET is:

\[
I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_p}\right)^2
\]

Where, \(I_{DSS}\) = max. drain current of a FET (i.e., when \(V_{GS} = 0\))

\(V_p\) = Pinch off voltage

Vin JFET

\[
V_{GS} = -ve \quad V_{DS} = +ve
\]

For n-channel JFET

\[
V_{GS} = \frac{V_p}{2} \quad V_{DS} = \frac{V_p}{2}
\]

Amplification factor \(\mu\) is given by

\[
\mu = \frac{r_d g_m}{V_{DS}}
\]

Where, \(g_m\) = trans-conductance

\(r_d\) = drain resistance

In JFET

\[
V_{GS} = -ve \quad V_{DS} = +ve
\]

For n-channel JFET
The basic principle of a MOSFET is “MOS capacitor”

In MOS capacitor has three operating modes
i. Accumulation mode
ii. Depletion mode
iii. Inversion mode

➢ In accumulation mode, the MOS capacitance is

\[
C_{MOS} = C_{ox} = \frac{\varepsilon_{ox} A}{t_{ox}}
\]

Where, \( \varepsilon_{ox} \) = permittivity of oxide which acts as dielectric

\[t_{ox} = \text{oxide layer thickness}\]

\[A = \text{Area}\]

➢ In depletion mode

\[
C_{MOS} = C_{ox} \parallel C_{dep} = C_{min}
\]

\[
C_{dep} = \frac{\varepsilon_{Si} A}{W_{dep}}, \quad C_{dep, min} = \frac{\varepsilon_{Si} A}{W_{d, max}}
\]

Therefore the MOS capacitance becomes minimum when it enters into depletion mode.
If $V_G$ is further increased, MOS capacitor will move into inversion mode.

In inversion mode,

$$C_{\text{MOS}} = C_\text{ox} = \frac{\varepsilon_\text{ox} A}{t_\text{ox}}$$

The C-V plot $C_{\text{MOS}}$

If $V_G$ increases continuously, the depletion layer width increases up to a maximum value and then becomes constant.

➢ Mathematical Analysis:

For a n-channel MOSFET.

$V_{GS} > V_{Th}$ – MOSFET is ON

$V_{GS} < V_{Th}$ – MOSFET is OFF

Where, $V_{Th} =$ Threshold voltage

If MOSFET is ON then it will be either in saturation region or in triode region.

$V_{DS} < (V_{GS} - V_{Th})$ – linear region

$V_{DS} \geq (V_{GS} - V_{Th})$ – saturation region

• Linear region:

$$I_D = \mu_n C_\text{ox} \frac{W}{L} \left[ (V_{GS} - V_{Th}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$r_D = \left[ \mu_n C_\text{ox} \frac{W}{L} (V_{GS} - V_T) \right]^{-1}$$

$$g_m = \mu_n C_\text{ox} \frac{W}{L} V_{DS}$$

Saturation region:

$$I_D = \mu_n C_\text{ox} \frac{W}{2L} [V_{GS} - V_{Th}]^2$$

$$r_{DS} = \frac{V_A}{I_D}$$

Where, $V_A =$ Early voltage

$$V_A = \frac{1}{\lambda}$$

Where, $\lambda =$ channel length modulation parameter ($V^{-1}$)

$$\therefore r_{DS} = \frac{1}{\lambda I_D}$$

The drain current including CLM is

$$I_D(\text{CLM}) = I_D \left( 1 + \lambda V_{DS} \right)$$

$$g_m = \mu_n C_\text{ox} \frac{W}{L} (V_{GS} - V_T)$$

(or)

$$g_m = \sqrt{2 \mu_n C_\text{ox} \frac{W}{L} I_D}$$

For FET,

$$V_{DS} \leq |V_P| - |V_{GS}| - \text{linear region}$$

$$\geq |V_P| - |V_{GS}| - \text{saturation region}$$

For p-channel MOSFET

$$V_{SG} > |V_P| - \text{MOSFET is ON}$$
The FET and MOSFET can be used as a voltage variable resistor in linear region (or triode region), and as an amplifier in saturation region.
The Threshold voltage is
+ve for n-channel EMOSFET
−ve for p-channel EMOSFET
In EMOSFET, the polarity of $V_{GS}$ is always opposite to the channel and substrate is also opposite to the channel.
The methods to reduce threshold voltage is
1. By reducing $t_{ox}$
2. By reducing the substrate concentration
3. Use of Si$_3$N$_4$ and SiO$_2$ instead of SiO$_2$
4. Poly crystalline silicon doped with boron instead of Aluminium
5. Silicon crystal with <100> orientation
6. Neglecting the body effect

Due to channel length modulation, the drain current in MOSFET increases and output resistance (drain resistance) decreases.

Due to body effect ($V_{SB} > 0$), the threshold voltage of a MOSFET increases which is avoidable. Therefore we neglect body effect.
1. **Series connection**

   ![Series Connection Diagram]

   \[ R_T = R_1 + R_2 \quad I_T = I_1 = I_2 \]
   \[ V_T = V_1 + V_2 \]

   i.e., In series connection voltage is different and current is same.

2. **Parallel connection**

   ![Parallel Connection Diagram]

   \[ R_T = \frac{R_1 \cdot R_2}{R_1 + R_2} \]
   \[ I_T = I_1 + I_2 \quad V_T = V_1 = V_2 \]

   i.e., In parallel connection current is different and voltage is same.

3. **Voltage division principle**

   ![Voltage Division Diagram]

   \[ V_o = \frac{R_2}{R_1 + R_2} \cdot V_{cc} \]

   Example:

   ![Example Diagram]

   \[ V_o = \frac{V_1 R_1 + V_2 R_1}{R_1 + R_2} \]

4. **Super position principle**

   ![Super Position Diagram]

   \[ V_o = \frac{V_1 R_1 + V_2 R_1}{R_1 + R_2} \]

5. **Nodal Analysis**

   (a) \[ V_1 \quad R \quad V_2 \]
   \[ I = \frac{V_1 - V_2}{R} \]
   \[ V_1 \quad R \quad -V_2 \]

   (b) \[ I = \frac{V_1 - (-V_2)}{R} \]

   (c) \[ \frac{V_1 - V_0}{R_1} + \frac{V_2 - V_0}{R_2} = 0 \]
(d) $V_1 \to V_2$

$I = C \frac{d(V_1 - V_2)}{dt}$

6. **Source transformation**

![Source transformation diagram]

$I_S \to V_S = I_S R_S$

7. **Current division principle:**

$I_T \to I_1$

$I_1 = \frac{R_2}{R_1 + R_2} \times I_T$

8. **Kirchhoff's Voltage Law:**

The algebraic sum of all the voltage drops around a closed loop is equal to zero.

Apply KVL around a loop

$10V - 4V - 2V - V_x = 0$

$V_x = 4V$

9. **Impedance calculations:**

$Z = R$

$Z = \frac{1}{sC}$

$Z = sL$

$z = R + sL$

$z = \frac{R \times \frac{1}{sC}}{R + \frac{1}{sC}}$

10. $Z = R + jX_L$

$|Z| = \sqrt{R^2 + X_L^2}$

$\angle Z = \tan^{-1}\left(\frac{X_L}{R}\right)$

**Diode Applications**

Diode equivalent circuit

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Forward Bias</th>
<th>Reverse Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Ideal diode</td>
<td>$V_{in} &gt; 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I = ?$</td>
</tr>
<tr>
<td>2.</td>
<td>Practical diode</td>
<td>$V_{in} &gt; V_f$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V = V_f$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I = ?$</td>
</tr>
</tbody>
</table>

The conditions for the forward bias of a p-n diode is

1. $V_{in} > 0 \rightarrow$ Ideal diode
VAN > V γ → Practical diode
2. V A > V K → Ideal diode
V A > (V K + V γ) → Practical diode

NOTE: V γ —— 0.7V —— Si
0.3V —— Ge

Comparison of Rectifiers:

<table>
<thead>
<tr>
<th>S. No</th>
<th>Name of the Rectifier</th>
<th>HWR</th>
<th>FWR</th>
<th>BR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>No. of diodes</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2.</td>
<td>I DC</td>
<td>I m / π</td>
<td>2I m / π</td>
<td>2I m / π</td>
</tr>
<tr>
<td>3.</td>
<td>V DC</td>
<td>V m / π</td>
<td>2V m / π</td>
<td>2V m / π</td>
</tr>
<tr>
<td>4.</td>
<td>I rms</td>
<td>I m / 2</td>
<td>I m / √2</td>
<td>I m / √2</td>
</tr>
<tr>
<td>5.</td>
<td>V rms</td>
<td>V m / 2</td>
<td>V m / √2</td>
<td>V m / √2</td>
</tr>
<tr>
<td>6.</td>
<td>γ</td>
<td>1.21</td>
<td>0.48</td>
<td>0.48</td>
</tr>
<tr>
<td>7.</td>
<td>η</td>
<td>40.6%</td>
<td>81.2%</td>
<td>81.2%</td>
</tr>
<tr>
<td>8.</td>
<td>TUF</td>
<td>0.286</td>
<td>0.69</td>
<td>0.81</td>
</tr>
<tr>
<td>9.</td>
<td>F</td>
<td>1.58</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>C</td>
<td>2</td>
<td>√2</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>PIV</td>
<td>V m</td>
<td>2V m</td>
<td>2V m</td>
</tr>
<tr>
<td>12.</td>
<td>f o</td>
<td>f in</td>
<td>2f in</td>
<td>2f in</td>
</tr>
<tr>
<td>13.</td>
<td>Conduction angle θ</td>
<td>π - 2 sin⁻¹ \left(\frac{V γ}{V m}\right)</td>
<td>2π - 4 sin⁻¹ \left(\frac{V γ}{V m}\right)</td>
<td>2π - 4 sin⁻¹ \left(\frac{2V γ}{V m}\right)</td>
</tr>
</tbody>
</table>

Some standard definitions:
- **Ripple Factor (γ):** The percentage of ac components present at the output of a rectifier. Usually the value of a ripple factor must be less.

\[ \gamma = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1} \]

- **% voltage Regulation:**

\[ \frac{R}{R_L} \times 100 \]

where \( R = R_F + R_{SW} \) —— HWR
\( = R_F + R_{SW} \) —— FWR
\( = 2R_F + R_{SW} \) —— B.R

Where
$R_F = \text{Forward bias resistance of a p-n diode}$

$R_{sw} = \text{DC resistance of Secondary winding}$

- **TUF**: It is the Ratio of dc output power and ac rating of secondary winding
  
  \[ \text{TUF} = \frac{P_{dc}}{P_{AC}} \times 100 \]

  
  \[ \therefore \text{The Higher TUF, then higher dc output power for given AC rating.} \]

- **Form factor (F)**: It is the ratio of rms output current (or) voltage to average (dc) value of output current (or) voltage
  
  \[ F = \frac{I_{rms}}{I_{DC}} \quad \text{or} \quad F = \frac{V_{rms}}{V_{dc}} \]

- **Crest Factor (C)**: It is the ratio of peak output current (or) voltage to rms value of output current (or) voltage
  
  \[ C = \frac{I_m}{I_{rms}} \quad \text{or} \quad C = \frac{V_m}{V_{rms}} \]

- **Efficiency (η)**: It is a measure of the ability of a rectifier to convert input power into dc power
  
  \[ \eta = \frac{\text{DC output power}}{\text{Input power}} \times 100 \]

- **Regulation Factor**: It is a measure of change in DC output voltage due to changes in the load current.

\[ \% \text{Regulation} = \frac{V_{DC(\text{NL})} - V_{DC(\text{FL})}}{V_{DC(\text{FL})}} \times 100 \]

**Note**: The ideal value of regulation factor is zero

- **Filters**:
  
  It is a circuit which eliminates the unwanted ac components and allows only dc components through it

- **Types of filters**:
  
  1. Capacitor Filter
  2. Inductor Filter
  3. LC Filter
  4. CLC Filter

**1. Capacitor Filter**:

(i) **Half wave Rectifier with C filter**

\[ V_{\gamma} = \frac{I_{dc}}{f_c} \]

\[ V_{dc} = V_m - \frac{V_{\gamma}}{2} \]

\[ \text{For HWR} \quad V_{dc} = V_m - \frac{I_{dc}}{2f C} \]

\[ \text{PIV} = 2V_m \]

\[ \gamma = \frac{1}{2\sqrt{3f C R_L}} \]

Therefore, the capacitor filter is suitable when $R_L$ is large (or) load current is small.

(ii) **Full wave Rectifier with C filter**

\[ V_{\gamma} = \frac{I_{DC}}{2f C} \]
- \[ V_{do} = v_m - \frac{I_{DC}}{4fC} \]
- \[ \gamma = \frac{1}{4\sqrt{3}fCR_L} \]

(2) Inductor Filter:

Full wave Rectifier with L filter

\[ \gamma = \frac{R_L}{3\sqrt{2}\omega_0L} \]

Half wave Rectifier

\[ \gamma = \frac{\pi R_L}{2\sqrt{2}\omega_0L} \]

\[ V_{dc} = \frac{2V_m}{\pi} - I_{dc}R \]

Where, \[ R = R_{sw} + R_F + R_{ind} \]

There the inductor filter is suitable for when \[ R_L \] is smaller (or) load current is large

LC filter (or) L-section filter

\[ \gamma = \frac{\sqrt{2}}{3} \frac{|X_C|}{|X_L|} \]  
FWR

Where \[ |X_C| & |X_L| \] should be calculated at \[ 2\omega_0 \]

\[ V_{dc} = \frac{2V_m}{\pi} - I_{dc}R \]

Where, \[ R = R_{sw} + R_F + R_{ind} \]

\[ \gamma = \frac{\pi}{2\sqrt{2}} \frac{|X_C|}{|X_L|} \]  

Where \[ |X_C| & |X_L| \] should be calculated at \[ \omega_0 \]

As the LC filter ripple factor is independent of \[ R_L \], it is suitable for any load.

In LC filter, inductance should be kept greater than critical inductance, so that peak ac current remains less than dc current.

CLC filter (or) \( \pi \)-filter

\[ \gamma = \frac{\sqrt{2}}{R_L} \frac{|X_C|^2}{|X_L|^2} \]

HWR & FWR

\[ |X_C| & |X_L| \] should be calculate at \( 2\omega_0 \) for FWR and \( \omega_0 \) for HWR.

\[ V_{DC} = V_m - \frac{V}{2} - I_{DC}R_{ind} \]

Zener diode:

Equivalent circuit

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Name of the component</th>
<th>DC (f = 0)</th>
<th>AC (f = \infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Resistor</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Capacitor $C$ | Open circuit | Short circuit
3. Inductor $L$ | Short circuit | Open circuit
4. Diode $V_D$ | $V_R$ | $r$

**Clippers:**

It is a non-linear wave shaping circuit

<table>
<thead>
<tr>
<th>Types of Clippers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series Clippers</td>
</tr>
<tr>
<td>Two level Clippers</td>
</tr>
<tr>
<td>Clipping using Zener diodes</td>
</tr>
<tr>
<td>Shunt Clippers</td>
</tr>
</tbody>
</table>

**Shunt Clippers:** If the input voltage and diode are connected in parallel, then the given clipper circuit is called shunt clipper.

Clipping above reference voltage

$$V_{in} < V_R, \ D-OFF, \ V_0 = V_{in}$$

$$V_{in} > V_R, \ D-ON, \ V_0 = V_R$$

Hence the output waveform is

The transfer characteristics are

- **Ideal Diode**
- **Practical Diode**
<table>
<thead>
<tr>
<th>S.No</th>
<th>Name of the Circuit</th>
<th>Output waveform</th>
<th>Transfer Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="Circuit Diagram" /></td>
<td><img src="image2" alt="Output Waveform" /></td>
<td><img src="image3" alt="Transfer Characteristic" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image4" alt="Circuit Diagram" /></td>
<td><img src="image5" alt="Output Waveform" /></td>
<td><img src="image6" alt="Transfer Characteristic" /></td>
</tr>
<tr>
<td>Diagram</td>
<td>Equation</td>
<td>Graph</td>
<td>Equation</td>
</tr>
<tr>
<td>--------</td>
<td>----------</td>
<td>------</td>
<td>----------</td>
</tr>
<tr>
<td><img src="image1" alt="Diag1" /></td>
<td>$V_R + 0.7V$</td>
<td><img src="image2" alt="Graph1" /></td>
<td>$V_R + 0.7$</td>
</tr>
<tr>
<td><img src="image4" alt="Diag2" /></td>
<td>$V_R - 0.7V$</td>
<td><img src="image5" alt="Graph3" /></td>
<td>$V_R - 0.7$</td>
</tr>
<tr>
<td><img src="image7" alt="Diag3" /></td>
<td>$V_R$</td>
<td><img src="image8" alt="Graph5" /></td>
<td>$V_R$</td>
</tr>
<tr>
<td><img src="image10" alt="Diag4" /></td>
<td>$V_{R1} - V_{R2}$</td>
<td><img src="image11" alt="Graph7" /></td>
<td>$V_{R2}$</td>
</tr>
<tr>
<td>Circuit Diagram</td>
<td>Waveform 1</td>
<td>Waveform 2</td>
<td>Waveform 3</td>
</tr>
<tr>
<td>-----------------</td>
<td>------------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td><img src="image1" alt="Circuit Diagram" /></td>
<td><img src="image2" alt="Waveform 1" /></td>
<td><img src="image3" alt="Waveform 2" /></td>
<td><img src="image4" alt="Waveform 3" /></td>
</tr>
<tr>
<td><img src="image5" alt="Circuit Diagram" /></td>
<td><img src="image6" alt="Waveform 1" /></td>
<td><img src="image7" alt="Waveform 2" /></td>
<td><img src="image8" alt="Waveform 3" /></td>
</tr>
<tr>
<td><img src="image9" alt="Circuit Diagram" /></td>
<td><img src="image10" alt="Waveform 1" /></td>
<td><img src="image11" alt="Waveform 2" /></td>
<td><img src="image12" alt="Waveform 3" /></td>
</tr>
<tr>
<td><img src="image13" alt="Circuit Diagram" /></td>
<td><img src="image14" alt="Waveform 1" /></td>
<td><img src="image15" alt="Waveform 2" /></td>
<td><img src="image16" alt="Waveform 3" /></td>
</tr>
<tr>
<td><img src="image17" alt="Circuit Diagram" /></td>
<td><img src="image18" alt="Waveform 1" /></td>
<td><img src="image19" alt="Waveform 2" /></td>
<td><img src="image20" alt="Waveform 3" /></td>
</tr>
</tbody>
</table>

**Legend:**
- $V_{in}$: Input voltage
- $R$: Resistor
- $Z_i$: Zener diode
- $V_{Zi}$: Zener voltage
- $V_i$: Output voltage
- $V_0$: Reference voltage

**Note:** The diagrams illustrate different configurations of a circuit using diodes and resistors, with accompanying waveforms for each configuration.
<table>
<thead>
<tr>
<th>6</th>
<th>$V_i$</th>
<th>$V_Z$</th>
<th>$V_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>$V_i$</td>
<td>$V_R$</td>
<td>$V_0$</td>
</tr>
<tr>
<td>8</td>
<td>$V_i$</td>
<td>$V_R$</td>
<td>$V_0$</td>
</tr>
<tr>
<td>Diagram</td>
<td>Description</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image1" alt="Diagram 1" /></td>
<td>Voltage divider with diode, input $V_{in}$, output $V_0$, resistance $R$, diode voltage $V_R$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image2" alt="Diagram 2" /></td>
<td>Voltage divider with diode, input $V_{in}$, output $V_0$, resistance $R$, diode voltage $V_R$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image3" alt="Diagram 3" /></td>
<td>Voltage divider with diode, input $V_{in}$, output $V_0$, resistance $R$, diode voltage $V_Z$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image4" alt="Diagram 4" /></td>
<td>Voltage divider with diode, input $V_{in}$, output $V_0$, resistance $R$, diode voltage $V_Z$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image5" alt="Diagram 5" /></td>
<td>Voltage divider with diode, input $V_{in}$, output $V_0$, resistance $R$, diode voltage $V_{i(max)}$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image6" alt="Diagram 6" /></td>
<td>Voltage divider with diode, input $V_{in}$, output $V_0$, resistance $R$, diode voltage $V_{i(max)}$.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
NOTE:

1. Here input is a AC sinusoidal waveform having 10V peak value.
2. \( V_R = \pm 5V \)
3. The condition to work as a two level clipper is \( V_{R1} > V_{R2} \)

- Clamper circuit is called DC restorer.
- **Voltage Multiplier Circuit:** A circuit whose input is an AC waveform of peak value \( V_m \) and output is a DC voltage which is an integral of peak input \( V_m \).
  
  \[
  V_0 = 2V_m \quad \text{Voltage doubler}
  \]
  
  \[
  = 3V_m \quad \text{Voltage tripler}
  \]
  
  \[
  = 4V_m \quad \text{Voltage quadrupler}
  \]

- **Voltage doubler:**

- **Voltage Tripler and Quadrupler:**

**Fig. Full-wave Voltage Doubler**

**Fig. Half-wave Voltage Doubler**
The factors that influence change in the collector current is

\[ I_c = f(I_{CO}, V_{BE}, \beta) \]

Therefore three are three stability factors

\[ S = \frac{\partial I_C}{\partial I_{CO}} \quad S' = \frac{\partial I_C}{\partial V_{BE}} \quad S'' = \frac{\partial I_C}{\partial \beta} \]

The ideal value of stability factor is zero.

**DC load line and Q-point:**

- AC load line is steeper then DC load line
- In order to maintain the stable Q-point, we follow some methods called stabilization methods
  - Stabilization methods
  - DC load line and Q-point:
  - Definition for Biasing:
    - The proper way of applying dc voltage (or) dc current to an electronic device (or) electronic circuit to operate it as desired.
    - The biasing methods are

**Fixed bias method**

![Fixed bias method diagram]

- The stability factor for a fixed bias method is \( S = 1 + \beta \)

**Collector to base bias method**

![Collector to base bias method diagram]

- In collector to base bias method, stability factor \( S \) depends on \( R_c \). If \( R_c \) is smaller (or) zero then collector current doesn’t remain stable.
- Resistance \( R_B \) causes negative feedback which reduces voltage gain.

**Voltage divider bias method:**

**Advantages:**

1. The stability factor \( S = 1 + \frac{R_B}{R_E} \)

   \[ \therefore \text{If } \frac{R_B}{R_E} \ll 1 \]

   then \( S = 1 \) which is excellent stability in \( I_c \) among all biasing circuits
   
   \( R_1 > R_2 \) otherwise battery life time gets reduced.
2. Stability factor is independent of $R_C$

3. It is used for any configuration (i.e. common Base, common collector and common emitter) Hence it is also called universal bias method

**Disadvantages:**

Resistance $R_E$ causes negative feedback which reduces voltage gain. This decrease in voltage gain can be prevented by connecting a large capacitor $C_E$ in parallel to $R_E$

**Current Mirror:**

The output current of a current mirror is

$$I_{out} = \frac{I_{in}}{1 + \frac{2}{\beta}}$$

$$\therefore I_{out} \approx I_{in}$$

**Compensation techniques:**

(i) for $I_{CO}$:

Here $R_T$ a thermistor which is a NTC device.

$T \uparrow \Rightarrow R_T \downarrow \Rightarrow V_E \uparrow \Rightarrow V_{BE} \downarrow \Rightarrow I_C \downarrow$

$\therefore$ As $I_C$ is increased due to temperature is reduced with the above compensation circuit.

**Thermal Runaway:**

- It is a process of self destruction (or) self damage of a transistor because of overheating.
- Thermal Resistance ($\theta$)

$$|\theta| = \frac{1}{\text{slope}} = \frac{T_{jmax} - T_A}{P_{Dmax}} \circ C/\text{watts}$$

Power derating curve of a BJT

- The condition to avoid thermal Runaway

$$\frac{\partial P_e}{\partial T_j} < \frac{\partial P_D}{\partial T_j}$$

$\text{or} \frac{\partial P_e}{\partial T_j} < \frac{1}{\theta}$

$\text{or} \ V_{CE} \leq \frac{V_{CC}}{2}$

**Small signal model:**

- Low frequency model
- High frequency model
- $h$-parameter model
- Hybrid $\pi$ - model
- Hybrid $T$ - model

- High frequency $\pi$-model
- High frequency $T$-model
### h- parameter model:

<table>
<thead>
<tr>
<th>S.No</th>
<th>Name of the parameter</th>
<th>Common emitter amplifier with bypass cap</th>
<th>Common collector amplifier</th>
<th>Common Base amplifier</th>
<th>Common emitter amplifier without bypass capacitor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Current gain ($A_1$)</td>
<td>$-h_{fe}$</td>
<td>$1 + h_{fe}$</td>
<td>$\frac{h_{fe}}{1 + h_{fe}} = 1$</td>
<td>$-h_{fe}$</td>
</tr>
<tr>
<td>2.</td>
<td>Input Resistance ($R_1$)</td>
<td>$h_{ie}$</td>
<td>$h_{ie} (1 + h_{fe}R_E)$</td>
<td>$\frac{h_{ie}}{1 + h_{fe}}$</td>
<td>$h_{ie} + (1 + h_{fe})R_E$</td>
</tr>
<tr>
<td>3.</td>
<td>Voltage gain ($A_v$)</td>
<td>$\frac{-h_{fe}R_L'}{h_{ie}}$</td>
<td>$(1 + h_{fe})R_E$</td>
<td>$\frac{h_{ie}R_L'}{h_{ie} + (1 + h_{fe})R_E} \approx 1$</td>
<td>$\frac{-h_{fe}R_L'}{h_{ie} + (1 + h_{fe})R_E}$</td>
</tr>
<tr>
<td>4.</td>
<td>Output Resistance ($R_o$)</td>
<td>$\frac{1}{h_{oe}}$</td>
<td>$\frac{R_S + h_{ie}}{1 + h_{fe}}$</td>
<td>$\frac{1 + h_{fe}}{h_{oe}}$</td>
<td>$\frac{1 + h_{fe}}{h_{oe}}$</td>
</tr>
<tr>
<td>5.</td>
<td>Phase shift between input and output</td>
<td>180°</td>
<td>0°</td>
<td>0°</td>
<td>180°</td>
</tr>
<tr>
<td>6.</td>
<td>Applications</td>
<td>Audio frequency amplifiers</td>
<td>Buffer in impedance matching applications</td>
<td>Radio freq. Amplifiers</td>
<td>Audio frequency amplifiers</td>
</tr>
</tbody>
</table>

- CE configuration has highest power gain among all configurations
- CE amplifier is not suitable for Radio (or) High frequency amplification because of the two reasons.
- Miller effect which causes increases in input capacitance.
- Internal capacitance of $J_c$ (collector base junction) can create positive feedback which can make the amplifier unstable at high frequency.
- CB amplifier does not have miller effect & internal positive feedback therefore it is appropriate for high frequency amplification.
Selection of an amplifier configuration for a cascade connection.

- Common collector stage —— $R_s = \text{High}$
- Common Base stage —— $R_s = \text{low}$
- The intermediates stages should be a common emitter stage
- The final stage should be common collector stage as it has a very low output resistance.

**Miller theorem:**

$$Z_1 = \frac{Z}{1 - A_v} \quad Z_2 = \frac{Z}{1 - \frac{1}{A_v}}$$

**Dual of Millers theorem:**

$$z_1 = z(1 - A_v) \quad z_2 = z(1 - \frac{1}{A_v})$$

**For a capacitor:**

$$C_1 = C(1 - A_v) \quad C_2 = C\left(1 - \frac{1}{A_v}\right)$$

**π-model of a BJT:**

**h-parameter Model:**

$$\beta = h_{fe} \quad r_\pi = h_{ie} \quad r_0 = \frac{1}{h_{oc}}$$

**Relation between π & h-parameters**
\[ \pi \text{ - parameters} \]

\[ g_m = \frac{I_c}{V_T} \]

\[ r_x = \frac{\beta}{g_m} \]  
(or)  
\[ r_x = \frac{V_T}{I_B} \]

\[ r_0 = \frac{V_A + V_{CE}}{I_C} \]

**NOTE:** The voltage gain of a CE amplifier with bypass capacitor is 

\[ A_V = -g_m R''_L \]

Where, \( R''_L = R_C || R_L || r_0 \)

**Low frequency analysis of an amplifier:**

| Large capacitor \((C_B, C_C, C_E)\) | Effect is considered | Short circuit | Short circuit |
| Small capacitor \((C_\pi, C_\mu)\) | Open circuit | Open circuit | Effect is considered |

\[ f_L = 3\text{dB cut off frequency} \]

\[ = \frac{1}{2\pi C_c (R_C + R_L)} \]  
if \( C_c \) is considered

\[ = \frac{g_m}{2\pi C_E} \]  
if \( C_E \) is considered

\[ = \frac{1}{2\pi C_B (R_s + R'_i)} \]  
if \( C_B \) is considered

- The overall \( f_L \) will be the highest of the above three frequencies if it higher by at least four times in comparison to other two frequencies.
- If no \( f_L \) is 4 times greater then

\[ f_L \cong 1.1 \times \sqrt{f_{L1}^2 + f_{L2}^2 + f_{L3}^2} \]

\[ |A_{vl}| = \frac{|A_{VM}|}{\sqrt{1 + (f_L/f)^2}} \]

\[ |A_{vl}| = 180^\circ + \tan^{-1}\left(\frac{f_L}{f}\right) \]
• High frequency analysis of BJT amplifier:
  - Hybrid (or) h-parameter model can’t be used in high frequency analysis because junction capacitances are not considered in it.
  - BJT amplifiers are analyzed at high frequencies using either high frequency π-model (or) high frequency T-model

➤ High frequency π-model:

- Short circuit current gain of CE configuration at High freq.
  \[ |A_{ie}| = \frac{\beta}{\sqrt{1+(f/f_\beta)^2}} \]
  \[ f_\beta = 3\text{dB cutoff frequency} \]
  \[ f_\beta = \frac{1}{2\pi(C_\pi + C_\mu) r_{b'e}} \]
- Unity gain frequency \( f_T \) of a BJT
  \[ f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \]
  \[ f_T = \beta f_\beta \]

• For FET
  \[ f_T = \frac{g_m}{2\pi(c_{gs} + c_{gd})} \]

- Since BJT has much higher trans conductance than FET gain band width product \( f_T \) is much greater for BJT than for FET.
- Hence BJT can act as better amplifier at high frequency in comparison to FET.

➤ High frequency π-parameters:

\[ g_m = \frac{I_c}{V_T} \]
\[ r_{b'e} = \frac{h_{fe}}{g_m} \]
\[ r_{bb'} = h_{ie} - r_{b'e} \Rightarrow r_{bb'}C_r = \frac{r_{b'C}}{h_{fe}} \]
\[ \frac{1}{f_{cc}} = g_{ce} = h_{oe} - \frac{1 + h_{fe}}{r_{b'C}} \]
\[ C_\pi + C_\mu = \frac{g_m}{2\pi f_T} \]

High frequency analysis of RC coupled amplifier

\[ f_H = \text{Upper 3 dB cutoff frequency} \]
\[ = \frac{1}{2\pi C_{sh} R_L'} R_L '' = R_C || R_L || r_{ce} \quad \text{if} \quad C_{sh} \]
\[ r = r_{bb'} || r_{b'e} \quad \text{if} \quad C_{in} \quad \text{is considered} \]
\[ = \frac{1}{2\pi C_{in} r} \]
The net (or) overall $f_H$ will be the smaller of the above two frequencies if it is smaller by at least four times.

If no frequency is 4 times smaller then

$$\frac{1}{f_H} = 1.1 \times \sqrt{\frac{1}{f_{H1}^2} + \frac{1}{f_{H2}^2}}$$

$$|A_{VH}| = \frac{A_{Vm}}{\sqrt{1 + (f/f_H)^2}}$$

$$|A_{VH}| = 180^0 - \tan^{-1}(f/f_H)$$

**Note:** RC couple common emitter amplifier provides phase shift of $180^0$ to medium frequencies greater than $180^0$ to low frequency and less than $180^0$ to High frequencies.

i.e, phase shift is $225^0$ $(180 + 45)$ @ $f_L$

$135$ $(180 - 45)$ @ $f_L$

Frequency response of RC coupled amplifier

Region (1):
- External capacitors ($C_B, C_c, C_E$) shows the effect on gain.
- Internal capacitors ($C_\pi, C_\mu$) becomes open circuit.

Region (2):
- Ext. capacitors – short circuit
- Internal capacitors – Open circuit

Region (3):
- External capacitors – Short circuit
- Internal capacitors – Shows the effect on gain.

**High frequency T – model:**

$$C_{jc} = \frac{\tau_1}{r_c}$$

Where $T_1 = \text{Transit time} = \frac{W_B^2}{2DB}$

$$C_{jc} = \frac{\varepsilon A}{w_d}$$

Short circuit current gain of $C_B$ configuration

$$|A_{ib}| = \frac{\alpha}{\sqrt{1 + (f/f_\alpha)^2}}$$

$$f_\alpha = \frac{1}{2\pi r_c C_{je}}$$

$$f_\alpha = \frac{DB}{\pi W_B^2}$$

$$f_\alpha = \frac{1}{2\pi T_1}$$

$$f_\alpha \approx (1 + \beta)f_\beta$$
Multistage amplifiers:

Cascading: It is a process of connecting the output of one amplifier as input to another amplifier.

Multistage amplifier (or) cascaded amplifier

Necessity of cascading:

1. To obtain higher amplification
2. To get the desired values of input and output resistances.

Effects of cascade:

1. Overall input resistance is same as input resistance of first amplifier
   \[ R'_1 = R'_{i1} \]
2. Overall output resistance is same as output resistance of last amplifier
   \[ R'_o = R'_{on} \]
3. Overall voltage gain is the product of individual voltage gains.
   \[ A_v = A_{v1} \times A_{v2} \times \cdots \times A_{vn} \]
   Overall voltage gain (in dB) is the sum of individual voltage gains (in dB)
   \[ A_v|_{dB} = A_{v1}|_{dB} + A_{v2}|_{dB} + \cdots + A_{vn}|_{dB} \]
4. Similarly current gain also
   \[ A_i = A_{i1} \times A_{i2} \times \cdots \times A_{in} \]
   \[ A_i|_{dB} = A_{i1}|_{dB} + A_{i2}|_{dB} + \cdots + A_{in}|_{dB} \]

Loading effect: The decrease in voltage gain of \( A_1 \) due to smaller input resistance of \( A_2 \) is known as loading effect.

If amplifiers having low (or) medium input resistance are cascaded, loading effect takes place and such amplifiers are called interacting amplifiers.

Eg: Cascade of CE amplifier (or) CB amplifier when amplifiers having high input resistance are cascaded, loading effect does not take place and such amplifiers are called Non interacting amplifiers.

Eg: Cascade of common source FET amplifiers.

(5) Cascading decreases bandwidth therefore cascading increases lower cut off frequency and decreases higher cut off frequency.

\[ (BW) = BW \times \sqrt{2^{1/n} - 1} \]

Where \( n = \) no. of stages
NOTE: For a cascade of interacting amplifiers, lower and higher cut off frequencies are calculated using

\[ f_L \approx 1.1 \times \sqrt{f_{L1}^2 + f_{L2}^2} \]

Where \( f_{L1}, f_{L2} \) etc are lower cut off frequencies of individual amplifiers.

\[ \frac{1}{f_H} \approx 1.1 \times \sqrt{\frac{1}{f_{H1}^2} + \frac{1}{f_{H2}^2}} \]

Where \( f_{H1}, f_{H2} \) etc are higher cut off frequencies of individual amplifiers.

» Special multistage amplifiers:

Darlington amplifier:

It has CC-CC configuration

Analysis

Input Res. \( R_i = (1 + h_{fe})^2 R_E \)

Output Res. \( R_o = \frac{h_{fe}}{1 + h_{fe} R_E} \)

Voltage gain \( A_v = 1 \)

\[ A_1 = \frac{(1 + h_{fe})^2}{1 + h_{oeb} h_{fe} R_E} \]

Overall \( \beta = \beta_1 + \beta_2 + \beta_2 \) (or) \( \beta \approx \beta_1 \beta_2 \)

Cascode amplifier:

It is cascade of CE and CB amplifiers (or) cascade of CS & CG amplifiers.

It can provide both voltage and current amplification hence it is preferred over single stage CB amplifier in high frequency amplification and wideband amplification.

» Condition for the region of operation in BJT is

\( I_C < \beta I_B \) - BJT is in saturation region.

\( I_C \geq \beta I_B \) - Active Region.

\( V_{CE} < V_{CE(sat)} \) - BJT is in saturation region.

\( V_{CE} > V_{CE(sat)} \) - BJT is in Active region.

» Power Amplifiers:

- It is a large signal amplifier
- In power amplifier, transistor behaves as non-linear element due to which harmonic (or) non-linear distortion can appear in the output. Therefore power amplifier is used as output stage (or) final stage in a multi stage amplifier.
Power amplifiers use a power transistor which has greater power rating. A power transistor has wider base in comparison to normal transistor wider base is helpful in rapid dissipation of heat to surroundings but $\beta$ becomes smaller due to greater no.of recombination’s.

Power transistor also use heat sinks for rapid dissipation of heat to surroundings, heat sink is made of alluminium which is good conductor of heat.

Performance of power amplifiers is measured by its conversion efficiency and figure of merit.

**Conversion Efficiency:**

$$\eta = \frac{\text{ac power supplied to load}}{\text{dc power taken from biasing supply}} \times 100$$

**Figure of Merit:**

$$F = \frac{\text{maximum power dissipation in the transistor}}{\text{maximum ac power supplied to the load}}$$

**Classification of Power Amplifier:**

Based on the location (or) position of operating point on the load line, a power amplifier can be of four types.

1. Class A power amplifier
2. Class B power amplifier
3. Class AB power amplifier
4. Class C power amplifier

**Class A Power Amplifier:**

- An amplifier in which operating point is located approximately at the centre of load line.

- Sinusoidal input can result in sinusoidal output hence output has least distortion, i.e., the conduction angle is $360^\circ$.

- Class A amplifier is used as small signal (or) voltage amplifier. It is not used as power amplifier because of greater quiescent power dissipation.

**Types of Class A Power Amplifier:**

1. Direct coupled amplifier
2. Transformer coupled amplifier
3. Class A Push-Pull amplifier
Direct Coupled Amplifier:

DC and AC load lines coincide for a direct coupled amplifier.

Conversion Efficiency:

\[ \% \eta = \left( 1 - \frac{V_{\text{min}}}{V_{\text{CC}}} \right) \times 25\% \]

Figure of Merit:

\[ F = 2 \]

Class A Transformer Coupled Amplifier:

Transformer coupling has following advantages.
- Maximum power can be supplied to the load due to impedance matching property of transformer.

Class B power Amplifier:

- An amplifier in which operating point is located at extreme ends load line i.e., operating point is located at either in saturation region (or) in cut-off region.

Note: DC load line for transformer coupled amplifier is a vertical line which cuts voltage axis at \( V_{\text{CC}} \).

\[ \% \eta = \frac{V_{\text{max}} - V_{\text{min}}}{V_{\text{max}} + V_{\text{min}}} \times 50\% \]

\[ \text{FOM} = 2 \]

Advantage: Quiescent power dissipation is negligible (or) zero.
**Drawback:** If input is sinusoidal, output signal will be half sinusoidal in class B amplifier. i.e., the conduction angle is $180^\circ$

- Therefore full sinusoidal output can be obtained in class B amplifier by using two transistors. Such circuit is called push-pull amplifier.
- Class B push-pull amplifier is used as untuned power amplifier.
- Untuned power amplifier is used for audio frequency amplification in public address system.

- **Types of Class B Push Pull amplifier:**
  1. Transformer coupled push-pull amplifier.
  2. Transformer less push pull amplifier.

- **Transformer Coupled Push-pull amplifier:**
  - Conversion Efficiency:
  \[
  \% \eta = \left( 1 - \frac{V_{\text{min}}}{V_{\text{max}}} \right) \times 78.5\%
  \]

  - Figure of Merit:
  \[
  F \approx 0.4
  \]

- **Advantages of class B over class A Power amplifier:**
  - Higher efficiency
  - Small figure of merit
  - Quiescent power dissipation is zero.

**Drawback:** Class B push-pull amplifier causes cross over distortion in the output waveform.

In order to overcome this, we use class AB push-pull amplifier.

\[
V_{\text{BE}} = 0 \text{ class B push-pull amplifier} = V_{\text{BE(cut-in)}} \text{ class AB push pull amplifier} = V_{\text{BE(active)}} \text{ class A push-pull amplifier}
\]

- Push-pull amplifier causes reduction in harmonic distortion by cancellation of even harmonic frequencies. (Which are more severe than odd harmonics)
- Push-pull output has half-wave symmetry.

- **Transformer less class B push-pull amplifier:**
  - As it is does not require any transformer it is appropriate for integrated circuits.
  - It uses a pair of complementary transistors i.e., one n-p-n transistor and one p-n-p transistor. Therefore it is also called complementary symmetry class B push-pull amplifier.
• Class C amplifier is always used as tuned power amplifier or radio frequency power amplifier.

➤ **Class AB Power Amplifier:**

• Amplifier in which operating point is located at a position which is other than class A and class B position.

• Performance lies between class A & class B amplifier.

• Output signal is more than half sinusoidal and not fully sinusoidal. Therefore the conduction angle is $180^\circ < \alpha < 270^\circ$.

• Class AB push-pull amplifier is also used as unturned (or) audio frequency power amplifier.

➤ **Differential Amplifier:**

• A circuit which amplifies the difference of two input voltages is called differential amplifier.

$$V_0 = A(V_1 - V_2)$$

Where, $A =$ Differential gain.

• A basic differential amplifier consists of two identical transistors connected as shown in figure.
• Since the two emitters are connected (or) coupled together it is also called emitter coupled differential amplifier.

• A differential amplifier can be used in four modes.
  1. Dual input balanced output mode
  2. Dual input unbalanced output mode
  3. Single input balanced output mode
  4. Single input unbalanced output mode

➢ Summery:

1. If output is balanced
   \[ A = -\frac{R_C}{r_e} = -g_mR_C \]
   
2. If output is unbalanced
   \[ A = -\frac{R_C}{2r_e} = -\frac{g_mR_C}{2} \]
   
i.e., \( A_{DM(bal)} > A_{DM(un bal)} \)

3. \( R_{i1} = R_{i2} = 2(1 + \beta)r_e = 2r_n = 2h_{ie} \)

4. \( R_{o1} = R_{o2} = R_C \)

5. \( A_{cm} = -\frac{R_C}{2R_E} \)

6. For a practical differential amplifier, output voltage can be expressed as
   \[ V_0 = A_{DM}V_d + A_{cm}V_{cm} \]
   Where, \( V_d = V_1 - V_2 \)
   \[ V_{cm} = \frac{V_1 + V_2}{2} \]
   \[ \therefore V_1 = \frac{V_d}{2} + V_{cm}, \quad V_2 = \frac{V_d}{2} - V_{cm} \]
   \[ V_0 = A_dV_d + A_{cm}V_{cm} \]
   \[ A_d = \frac{A_1 - A_2}{2} \]
   \[ A_{cm} = A_1 - A_2 \]

In a differential amplifier, the undesired output should as small as possible. Therefore, \( A_{cm} \) should be made smaller by increasing \( 'R_E' \)

7. CMRR

\[ CMRR = \left| \frac{A_{dm}}{A_{cm}} \right| \]

It is the measure of the ability of a differential amplifier to reject common mode input.

In a ideal differential amplifier, \( A_{cm} = 0 \) & \( CMRR = \infty \)

\[ CMRR = 2g_mR_E \text{ for balanced output} \]
\[ = g_mR_E \text{ for unbalanced output} \]

Therefore if a differential amplifier is fabricated on a silicon chip, \( R_E \) can be
replaced with a current mirror which results in following advantages.

1. $A_{cm}$ decreases (or) CMRR increases because current mirror has high output resistance.
2. Smaller chip area is required
3. Smaller supply voltage is needed
4. As current mirror is a constant current source circuit, it helps in maintaining the dc collector currents of $Q_1$ and $Q_2$ stable.

**FET & MOSFET Analysis**

- The Q-point parameters in JFET are $I_D$ and $V_{GS}$.
- In JFET’s as input junction is reverse biased.
  Hence, $I_G = 0$
  \[ \therefore I_D = I_S \]
- If the two identical JFET’s are connected in parallel then overall
  \[ g_m' = 2g_m \]
  \[ r_d' = \frac{r_d}{2} \]
- If non-identical then
  \[ \mu' = \frac{\mu_1 r_d_2 + \mu_2 r_d_1}{r_d_1 + r_d_2} \]

**Biasing Methods of JFET and MOSFET:**

1. **Gate bias circuit:**

   \[ V_{GS} = -V_{GG} \]
   \[ I_D = I_{DSS} \left[ 1 - \frac{V_{GS}}{V_P} \right]^2 \]
   \[ V_{DS} = V_{DD} - I_D R_D \]
   $R_D$ should be selected such that $V_{DS} > |V_P| - |V_{GS}|$. Thus JFET operates in saturation region.

   **NOTE:** The drawback of gate bias method is it requires two biasing supplies

2. **Source self bias circuit:**

   \[ V_{GS} = -I_D R_S \]
   \[ I_D = I_{DSS} \left[ 1 - \frac{V_{GS}}{V_P} \right]^2 \]
   \[ V_{DS} = V_{DD} - I_D R_D \]
Voltage drop across $R_s$ causes reverse biasing of gate channel junction. This technique of maintaining gate channel junction in reverse bias through voltage drop across $R_s$ is known as self biasing.

$\therefore R_s$ is a self bias resistor.

**Advantages:**
- It requires one biasing supply
- It maintains drain current stable

If $I_{DS} \uparrow \Rightarrow V_{GS}$ becomes more negative $\Rightarrow$ channel width $\downarrow \Rightarrow R_{DS} \uparrow \Rightarrow I_{DS} \downarrow$

This increase and decrease in $I_{DS}$ will cancel each other and thus current remains stable

**NOTE:** In a JFET, the decrease in $I_D$ is 0.7% for $1^\circ C$ rise in temperature

**Disadvantages:**
- $R_s$ causes negative feedback leads to decreases in voltage gain
- It can be prevented by connecting a large capacitor across it.

**(3) Voltage divider bias method:**

$V_{GS} = \frac{V_{DD}R_2}{R_1 + R_2} - I_D R_S$

$I_D = I_{DSS} \left[ 1 - \frac{V_{GS}}{V_P} \right]^2$

$V_{DS} = V_{DD} - I_D (R_D + R_S)$

Source self bias circuit is preferred to bias JFET over voltage divider bias method since later requires more no. of resistors.

**Important points:**
- Depletion MOSFET can work for any $V_{GS} [0, +V_e / -V_e]$ Hence any of the above three biasing circuits can be used to bias D MOSFET.

n-channel EMOSFET requires $+Ve V_{GS}$ therefore self bias circuit can’t be used.

For a JFET (or) MOSFET, the operating point is are $V_{DS}, I_{DS}$

**Small signal model of FET:**

$$I_{DS} = g_m V_{gs} + \frac{1}{r_{ds}} V_{ds}$$

**Small signal analysis of FET amplifiers**
- Analysis is similar for JFET & MOSFET amplifiers.
Common source amplifier with bypass capacitor.

- Since input current, $I_G = 0$ CS amplifier can’t be used for current amplification.
- Input resistance ($R_i$) $= \infty$
  \[ R_i = \infty \parallel R_G \quad R_i^1 = R_G \]
- Voltage gain ($A_v$) $= -g_m\left( r_{ds} \parallel R_L \right)$
- For common emitter and common source amplifier voltage gain expression is similar. Since BJT has higher trans-conductance, voltage gain of CE amplifier will be much greater than CS amplifier.
- Output resistance ($R_o$)
  \[ R_o = r_{ds} \quad R_o^1 = r_{ds} \parallel R_D \]

Common drain amplifier:

- Input resistance ($R_i$)
  \[ R_i = \infty \quad R_i^1 = \infty \parallel R_G \]
  \[ R_i^1 = R_G \]
- Voltage gain ($A_v$)
  \[ A_v = \frac{g_m (R_s \parallel r_{ds})}{1 + g_m (R_s \parallel r_{ds})} \]
  \[ A_v \approx 1 \]
- Hence common drain amplifier is known as source follower. Therefore it is used as buffer.
- Output resistance ($R_o$)
  \[ R_o = r_{ds} \parallel \frac{1}{g_m} \quad R_o^1 = r_{ds} \parallel \frac{1}{g_m} \parallel R_s \]

Common gate amplifier:
- Current gain \( A_I \)
\[
A_I = \frac{1}{g_m}
\]

- Input resistance \( R_i \)
\[
R_i = \frac{1}{g_m}
\]
\[
R_i^I = R_s \| \frac{1}{g_m}
\]

- Voltage gain \( A_v \)
\[
A_v = g_m R_L^I
\]

- Output resistance \( R_o \)
\[
R_o = \infty \quad R_o^I = R_D
\]

**Common source amplifier with bypass capacitor:**

- Input resistance
\[
R_i = \infty \quad R_i^I = R_G
\]

- Voltage gain
\[
A_v = \frac{-g_m R_L^I}{1 + g_m R_s}
\]

- Output resistance
\[
R_o = r_d + (1 + \mu) R_s
\]
\[
R_o^I = R_o \| R_D
\]

- If bypass capacitor is disconnected \( R_i \) & \( R_o \) will increase whereas \( |A_v| \) decreases.

**Feedback Amplifiers**

Drawbacks of basic amplifiers:

(i) Gain is unstable due to variations in transistor parameters such as temperature and ageing effect.

(ii) Distortion in the output

(iii) Input and output resistances are not desired feedback amplifier

Feedback is a process of mixing a part of the output with applied input.

**Types of feedback**

- Positive feedback
  \[ x_i = x_s + x_f \]
  *Eg*: Oscillators, multivibrators

- Negative feedback
  \[ x_i = x_s - x_f \]
  *Eg*: Amplifiers
Effects of Negative feedback on characteristics of amplifier:

1. Decreases the gain
   \[ A_f = \frac{A}{1 + A\beta} \]

2. Increases band width
   \[ (BW)_{FBA} = (BW)_{BA} (1 + A\beta) \]

3. Stabilizes the gain
   \[ \frac{dA_f}{A_f} = \frac{1}{1 + A\beta} \times \frac{dA}{A} \]

4. Harmonic distortion and frequency distortion reduces.

5. Input and output resistances

<table>
<thead>
<tr>
<th>S. No</th>
<th>Name of the feedback</th>
<th>( R_{if} )</th>
<th>( R_{of} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Voltage series</td>
<td>( R_i (1 + A\beta) )</td>
<td>( \frac{R_o}{1 + A\beta} )</td>
</tr>
<tr>
<td>2.</td>
<td>Voltage shunt</td>
<td>( \frac{R_i}{1 + A\beta} )</td>
<td>( \frac{R_o}{1 + A\beta} )</td>
</tr>
<tr>
<td>3.</td>
<td>Current series</td>
<td>( R_i (1 + A\beta) )</td>
<td>( R_o (1 + A\beta) )</td>
</tr>
<tr>
<td>4.</td>
<td>Current shunt</td>
<td>( \frac{R_i}{1 + A\beta} )</td>
<td>( R_o (1 + A\beta) )</td>
</tr>
</tbody>
</table>

- Amount of negative feedback is \( 20 \log_{10} (1 + A\beta) \)

- Sensitivities is a measure of how sensitive the gain is \( S = \frac{1}{1 + A\beta} \)

- Smaller sensitivity indicates better stability in gain de-sensitivity (or) Return difference
  \[ D = \frac{1}{S} \quad D = 1 + A\beta \]

Types of Basic amplifiers:

<table>
<thead>
<tr>
<th>S. No</th>
<th>Name of the Basic amp</th>
<th>( R_{if} )</th>
<th>( R_{of} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Voltage amplifier</td>
<td>( \infty )</td>
<td>0</td>
</tr>
<tr>
<td>2.</td>
<td>Current amplifier</td>
<td>0</td>
<td>( \infty )</td>
</tr>
<tr>
<td>3.</td>
<td>Trans resistance amplifier</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4.</td>
<td>Trans conductance amplifier</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>
- The Basic amplifier is Trans resistance amplifier.
- Also called shunt-shunt feedback amplifier (or) voltage current feedback amplifier.

- The Basic amplifier is Voltage amplifier.
- Also called series-shunt feedback amplifier (or) voltage feedback amplifier.

- The Basic amplifier is Transconductance amplifier also called series-series feedback amplifier (or) current voltage feedback amplifier.
- The Basic amplifier is current amplifier.
- Also called current-current feedback amplifier (or) shunt series feedback amplifier.
- Collector to base bias method has voltage shunt feedback.
- Common emitter (or) common source amplifier without bypass capacitor has current series feedback amplifier.
- Common collector amplifier has voltage series feedback.
- Common collector (or) common source amplifier with bypass capacitor has no feedback.
- A circuit may have multiple feedback based on the requirement, we can eliminate the unnecessary feedback.
- A circuit may have both global feedback and local feedback. In this case, we need to consider global feedback only as it is dominant.
Oscillators

- A circuit which can generate AC waveform without using ac input is called oscillator.
- Oscillator uses positive feedback and generates sinusoidal waveforms.
- A circuit which generates non sinusoidal waveforms such as triangular wave form, square waveform are known as wave from generators.

➢ Condition for oscillator:
Also called as Barkhausen criteria.

(i) $|A\beta| = 1$

(ii) $\angle A\beta = 0^\circ (or) 360^\circ (or) 2\pi n$ n-integer

➢ Types of oscillators:

Types of oscillators

- low frequency oscillator (or)
- High frequency oscillator (or)
- Audio frequency oscillator
- Radio frequency oscillator
- RC phase shift OSC
- wein brige OSC
- LC OSC
- Crystal OSC
- Hartley OSC
- Collpids OSC
- Clapp OSC
- Using FET
- Using op-amp
- Using BJT

RC phase shift oscillator
Using JFET

- It has two sections common source amplifier and RC feedback network
- Here common source amplifier provides phase shift of $180^\circ$ and RC feedback network generates additional phase shift of $180^\circ$.
- Thus total phase shift of feedback loop becomes $360^\circ$ and behaves as oscillator.

$$f_0 = \frac{1}{2\pi RC\sqrt{6}}$$

$|A| > 29$

- Using op-amp

$$f_0 = \frac{1}{2\pi RC\sqrt{6}}$$

$|A| \geq 29$
RC phase shift Oscillator:

\[ f_0 = \frac{1}{2\pi RC \sqrt{6+4K}} \]

\[ K = \frac{R_C}{R} \]

Condition for sustained output is

\[ h_{fe} \geq 23 + \frac{29}{K} + 4K \]

\[ h_{fe} \geq 44.5 \]

Wien bridge Oscillator:

\[ f_0 = \frac{1}{2\pi RC} \]

\[ |A| \geq 3 \]

LC oscillators:

It is used as RF oscillator

Types:
1. Hartley Oscillator
2. Colpitts oscillator
3. Clapp oscillator

Hartley Oscillator:

- Here \( C_B \) and \( C_C \) prevent dc grounding of base and collector through \( L_1 \) and \( L_2 \) respectively.

\[ f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2)C}} \]

- If mutual inductance considered

\[ f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2 + 2M)C}} \]

- The condition for sustained output

\[ h_{fe} \geq \frac{L_2}{L_1} \frac{\frac{L_1}{L_2}} {\frac{1}{R_C} h_{fe}} - \text{BJT Oscillator} \]

\[ |A| \geq \frac{L_2}{L_1} \]
Colpitts Oscillator:

\[ f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{L \left( \frac{1}{C_1} + \frac{1}{C_2} \right)}} \]

Condition for sustained output

\[ h_{fe} \geq \frac{C_2}{C_1} + \frac{C_1 h_{fe}}{C_2 R_C} \quad \text{- BJT Oscillator} \]

\[ |A| \geq \frac{C_1}{C_2} \quad \text{- FET Oscillator} \]

**Drawback:** \( f_0 \) is unstable due to the effect of internal input and output capacitances of BJT (or) FET. Since input, output capacitances vary with time and temperature.

In order to overcome this, a small value of capacitor \( C \) can be connected in series with inductor \( L \), such circuit is called clap oscillator.

\[ f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{L \left( \frac{1}{C} \right)}} \]

\[ \therefore \text{In clap oscillator, } f_0 \text{ is independent of input, output capacitances.} \]

Crystal Oscillator:

It is an LC oscillator in which a quarter crystal is used in place of inductor.

The symbol of crystal is

\[ \square \]

The equivalent circuit is
Properties of Crystal Material:

1. At \( f = f_s \), \( Z = 0 \) i.e., at \( f_s \) crystal undergoes series resonance. Therefore \( f_s \) is called series resonant frequency.

\[
f_s = \frac{1}{2\pi} \sqrt{\frac{1}{L} \cdot \frac{1}{C}}
\]

2. At \( f = f_p \), \( Z = \infty \) i.e., at \( f_p \) quartz crystal undergoes parallel resonance. Therefore, \( f_p \) is called parallel resonant frequency.

\[
f_p = \frac{1}{2\pi} \sqrt{\frac{1}{L} \left( \frac{1}{C} + \frac{1}{C'} \right)}
\]

3. From the above two equations

\( f_p > f_s \)

4. If \( f_s < f < f_p \), \( z = +jx \)

Due to positive reactance a quartz crystal behaves as inductor. If the frequency of operation lies between \( f_s \) and \( f_p \), Hence it can be used as an inductor in LC oscillator.

\[ f_0 = f_p \]

Since properties of quartz material do not change with time & temperature, its internal inductance and capacitance remains stable.

Drawbacks:
1. It is expensive
2. \( f_0 \) can’t varied.

Application:
1. \( \mu_p, \mu_c \)
2. Watches
3. Local oscillator in receivers

Operational Amplifiers

- It is a high voltage gain direct coupled amplifier
- It can be used to perform mathematical operations on analog signals therefore it is called operational amplifier.
- Op-amp is available as IC 741 in 8 pin package.

Logic Diagram

Offset Null

Inv. and Non inv.

Output

\( V_{CC} \)

\( V_{EE} \)
Properties of op-amp:

<table>
<thead>
<tr>
<th>S.No</th>
<th>Name of the parameter</th>
<th>Ideal</th>
<th>Practical</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Open loop gain ( A_{OL} )</td>
<td>( \infty )</td>
<td>( 10^6 )</td>
</tr>
<tr>
<td>2</td>
<td>Input Resistance ( R_i )</td>
<td>( \infty )</td>
<td>1MΩ to 2MΩ</td>
</tr>
<tr>
<td>3</td>
<td>Output Resistance ( R_o )</td>
<td>0</td>
<td>50Ω to 100Ω</td>
</tr>
<tr>
<td>4</td>
<td>Open loop ( f_{OL} )</td>
<td>( \infty )</td>
<td>5Hz</td>
</tr>
<tr>
<td>5</td>
<td>Offset</td>
<td>Yes</td>
<td>Non-zero</td>
</tr>
<tr>
<td>6</td>
<td>CMRR</td>
<td>( \infty )</td>
<td>( 10^6 ) (or) 120dB</td>
</tr>
<tr>
<td>7</td>
<td>Slew rate</td>
<td>( \infty )</td>
<td>0.5V/μsec to 1V/μsec</td>
</tr>
</tbody>
</table>

Note:
1. We can increase the bandwidth by applying negative feedback to the op-amp.
2. Slew rate is expressed in \( V/\mu\text{sec} \).

- The condition to avoid distortion due to slew rate is
  \[
  \frac{2\pi A_{CL} V_{sat} f_0}{10^6} \leq SR
  \]
- The VGB (unity gain bandwidth product) of 741 is
  \[
  \text{VGB} = A_{OL} \times f_{OL}
  \]
  \[
  = 10^6 \times 5 = 5 \text{ MHz}
  \]
- The transfer characteristics of ideal op-amp:
  
  It follows the signum function.
  The transfer characteristics of practical op-amp is

- When op-amp is used without feedback and with positive feedback difference input \( V_d \) will be large enough therefore op-amp works in saturation region.
- If op-amp is used with negative feedback, the difference input becomes smaller because feedback signal opposes the applied input. As a result, op-amp works in linear region.
- In negative feedback circuits, virtual short circuit condition and virtual ground condition is satisfied.
  i.e., \( V_1 \approx V_2 \)
Characteristics of Non-ideal op-amp:

1. Input bias current: It is the average of 2 input currents of op-amp.
   \[ I_B = \frac{I_{B_1} + I_{B_2}}{2} \]
   It lies in 100’s of nA.

2. Input offset current: It is the difference of two input currents.
   \[ I_{io} = |I_{B_1} - I_{B_2}| \]
   It lies in nA.
   Note: \( I_B > I_{io} \)

3. Output offset voltage (\( V_{oo} \)): It is the output voltage of op-amp when \( V_1 = 0 \) and \( V_2 = 0 \)

Application of op-amp:

1. Open loop applications:
   Voltage comparator

   It is used as a zero crossing detector.
   For a zero crossing detector the output will be square wave.
   The zero crossing detector can not give square wave output for any other arbitrary waveform which is not sinusoidal. To avoid this draw back we have a special circuit called as “Schmitt Trigger”.

2. Negative feedback applications:

   1. Non-inverting amplifier:

      \[ V_0 = \left( 1 + \frac{R_F}{R_1} \right) V_{in} \]
      \[ R_{if} = \infty \]
      \[ R_{of} \approx 0 \]

      It has voltage series feedback.

   2. Inverting amplifier:

      \[ V_0 = -\frac{R_F}{R_1} V_{in} \]
3. Voltage follower:

\[ V_0 = V_{in} \]

It is a unity gain non inverting amplifier

\[ R_{if} = \infty, \ R_0 = 0 \]

It has voltage series feedback

Bandwidth = \( U\text{GB} \)

**Applications:**
1. Used as a buffer
2. Used in sample and hold circuit
3. Used in instrumentation amplifier

4. Differential amplifier:

\[ V_0 = \frac{R_2}{R_1} (V_1 - V_2) \]

\[ R_{i1} = R_1 + R_2, \ \ R_{i2} = R_1 \]

\[ R_{id} = 2R_1, \ R_{of} = 0 \]

Effect of input bias current

Let \( I_B = 500\text{nA}, \ R_F = 100\text{M} \Omega \)

\[ \therefore V_0 = I_B R_F = 5\text{V} \]

- Output offset voltage due to input bias current is considerable in inverting amplifier where \( R_f \) is large and it can be neglected in non-inverting amplifier, when \( R_f \) relatively smaller.
- Output offset voltage can be reduced to zero by connecting a compensation resistance from non-inverting node to ground.

\[ R_{comp} = R_F || R_1 \]

- \( R_{comp} \) is necessary in inverting amplifier and it may (or) may not be used in non-inverting amplifier.

5. Inverting amplifier:

\[ V_0 = \frac{-R_F}{R} (V_1 + V_2 + V_3) \]
6. Non-inverting summing amplifier:

\[ V_0 = V_1 + V_2 + V_3 \]

7. Adder/subtractor:

\[ V_0 = (V_3 + V_4) - (V_2 + V_1) \]

8. Current to voltage converter:

\[ V_0 = -R_F I_S \]

In general it has is trans-resistance amplifier.

9. Voltage to current converter:

\( I_L = \frac{V_S}{R} \)

In general, it is transconductance amplifier.

(1) Floating load

(2) Grounded load

\[ I_L = \frac{-V_S}{R_2} \]

10. Instrumentational amplifier:

The two voltage followers prevent loading of the bridge i.e., they prevent current flow out of the bridge.
11. Log amplifier:

\[ V_0 = -\eta V_T \ln \left( \frac{V_i}{I_0 R} \right) \]

Log amplifier can also be formed by using a transistor in place of the diode.

12. Anti-log amplifier:

\[ V_0 = -I_0 R_F e^{-\eta VT} \]

Anti log amplifier can also be formed by using a transistor in place of the diode.

13. Analog multiplier:

If adder is replaced by a subtractor, circuit becomes divider.

14. Precision diode (Super diode):

A precision diode conducts when \( V_{in} > 0 \) and remains OFF when \( V_{in} < 0 \)

With \( \mu \text{V} \) or \( \text{mV} \), we can conduct the precision diode.

15. Precision Half wave rectifier:

- It is used to rectify weak ac signals.
- A precision rectifier is used in biomedical engineering to rectify human body potential which lie in \( \mu \text{V} \) or \( \text{mV} \) for medical diagnosis, where as a normal rectifier is used in power supplies to convert ac power into dc power.
- During positive half cycle
  \[ D_1 = \text{OFF}, \ D_2 = \text{ON} \]
  \[ \therefore V_0 = 0 \]
- During negative half cycle
  \[ D_1 = \text{ON}, \ D_2 = \text{OFF} \]
  \[ V_0 = -V_i \]
- The transfer characteristics are as follows

![Graph showing transfer characteristics](image)

Here, \( D_1 \) – rectification
\( D_2 \) – prevents saturation of op-amp.

16. **Precision Full wave rectifier:**

![Diagram of a precision full wave rectifier](image)

- During positive half cycle,
  \[ D_1 = \text{ON}, \ D_2 = \text{OFF} \]
  \[ \therefore V_0 = V_i \]
- During negative half cycle,
  \[ D_1 = \text{OFF}, \ D_2 = \text{ON} \]
  \[ \therefore V_0 = -V_i \]

- The transfer characteristics are as follows

![Graph showing transfer characteristics](image)

17. **Integrator:**

![Diagram of an integrator](image)

\[ V_0 = -\frac{1}{RC} \int V_{in} \, dt \]

<table>
<thead>
<tr>
<th>( V_i )</th>
<th>( V_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step input</td>
<td>Ramp output</td>
</tr>
<tr>
<td>Impulse input</td>
<td>Step output</td>
</tr>
<tr>
<td>Ramp input</td>
<td>Saw tooth output</td>
</tr>
</tbody>
</table>

Integrator becomes unstable at very low frequencies. To overcome this drawback of instability, a resistor can be connected in parallel to \( C_F \) such circuit is called practical integrator.

![Diagram of a practical integrator](image)

The 3dB cut-off frequency is
\[ f = \frac{1}{2\pi R_F C_F} \]
18. Differentiator:

\[ V_0 = -R_F C \frac{dV_i}{dt} \]

The differentiator becomes unstable at very high frequencies. To overcome this drawback of instability, a resistor \( R \) can be connected in series with a capacitor such circuit is called a practical differentiator.

19. Band Pass Filter:

\[ f_H = \frac{1}{2\pi R_F C_F}, \quad f_L = \frac{1}{2\pi RC} \]

RC elements should be selected such that \( f_H \) much greater than \( f_L \) only then circuit behaves as band pass filter otherwise the circuit becomes band stop filter.

Active Filters:
- It consists of op-amp & RC elements.
- RC elements perform filtering & op-amp provides amplification.
- Inductors are not preferred in active filters because of bulky is size.

20. Low pass Filter:

\[ |A_{CL}| = \frac{A_{\text{max}}}{\sqrt{1 + \left( \frac{f}{f_c} \right)^2}} \]

Where, \( A_{\text{max}} = 1 + \frac{R_2}{R_1} \)

\[ \angle A_{CL} = -\tan^{-1} \left( \frac{f}{f_c} \right) \]

\[ f_c = \frac{1}{2\pi RC} \]

The Roll-off rate is 20 dB/dec.

Frequency response can be improved & roll-off rate can be increased by increasing the order of the filter.

21. High pass Filter:
\[ |A_{CL}| = \frac{A_{\text{max}}}{\sqrt{1 + \left(\frac{f_c}{f}\right)^2}} \]
\[ \angle A_{CL} = \tan^{-1}\left(\frac{f_c}{f}\right) \]

22. **Band Pass Filter:**

It is a cascade of LPF & HPF.

\[ \text{Circuit behaves BPF only if LPF cut off frequency is much greater than HPF cut off frequency.} \]

**Note:** \( f_H < f_L \)

23. **Band Stop Filter:**

\[ \text{Circuit behaves BSF only if HPF cut-off frequency is much greater than LPF cut-off frequency.} \]

**Note:** \( f_H > f_L \)

24. **All Pass Filter:**

It passes all frequencies but provides unequal phase shift to different frequencies present in the input signal.

It is used for phase equalization (or) delay equalization.

The transfer function of All Pass Filter is

\[ A_{CL} = \frac{1 - sRC}{1 + sRC} \]
\[ |A_{CL}| = 1 \]
\[ \angle A_{CL} = -2\tan^{-1}(\omega RC) \]

Since \( \omega \) lies between 0 and \( \infty \), phase shift of all pass filter lies between 0 and \(-180^\circ\).

25. **Voltage Limiters:**

Voltage limiters are used to limit a part of the signal input \( V_i \)

The output \( V_0 \) is
The output $V_0$ is

$$\left( V_z + V_i \right)$$

$-V_{sat}$

The output $V_0$ is

$$\left( V_{z1} + V_{z2} \right)$$

$$-\left( V_{z1} + V_{z2} \right)$$

26. Peak Detector:

$V_i$

$V_{m1}$

$V_{m2}$

$V_{m3}$

- $V_0 = V_{m3}$

- With finite gain (non ideal) op-amp
  Non-inverting amplifier:

$$A_{CL} = \frac{A_{OL}}{1 + A_{OL}\beta}$$

Where, $\beta = \frac{R}{R + R_f}$

The above circuit has voltage series feedback which increases input resistance and decreases output resistance.

$$R_{if} = R_i (1 + A_{OL}\beta)$$

$$R_{if} = \frac{R_0}{(1 + A_{OL}\beta)}$$

Bandwidth $= \frac{UGB}{A_{CL}}$

Inverting Amplifier:
\[ A_{CL} = \frac{-A_{OL}}{1 + A_{OL} \beta} \times K \quad K = \frac{R_F}{R + R_F} \]

\[ R_{if} = R + (R_m || R_i) \]

Where, \( R_m = \frac{R_F}{1 + A_{OL}} \)

\[ R_{of} = \frac{R_0}{1 + A_{OL} \beta} \]

\[ BW = (1 + A_{OL} \beta) \times \text{open loop BW} \]

\[ BW = \frac{UGB}{A_{CL}} \times K \]

**Note:** If op-amp has finite open loop gain then property of virtual short circuit can’t be used.

**Positive Feedback Applications:**

**Schmitt Trigger:**

- It is a comparator circuit in which positive feedback (or) regenerative feedback is used. Therefore Schmitt trigger is also called “regenerative comparator”.
- It convert any waveform into square wave. Therefore it is also called squaring circuit.

- The transfer characteristics are

\[ V_H = V_{UTP} - V_{LTP} \]

- Reference voltage can be used in a Schmitt trigger to change threshold voltages.
- \( V_H \) is independent of \( V_{ref} \)
- For a inverting type Schmitt trigger, the hysteresis loop in clockwise direction.
- If two Zener diodes are connected back to back they will perform two level clipping. As a result output waveform, \( V_0 \) gets limited to \( V_{Z1} \) and \(-V_{Z2}\)
For non-inverting type, Schmitt trigger, transitions occur anti clock wise in transfer characteristics.

Astable Multivibrator:
Also called as square wave generator (or) free running oscillator.

Since capacitor charges and discharges through same resistor, the output waveform has 50% duty cycle.

Monostable Multivibrator:

Duty cycle other than 50% can be obtained by replacing resistor R with a network as shown.
**Voltage Regulators**

- The basic element in a voltage regulator is Zener diode.
- Zener diode acts as constant voltage source in beyond breakdown region.
  \[
  V \geq V_Z \\
  I_{ZK} < I < I_{Z_{max}}
  \]

\[
V_Z \\
I_{ZK} \\
I_{Z_{max}}
\]

Zener diode shunt regulator

\[
V_0 = V_Z \\
I_S = \frac{V_S - V_Z}{R_S} \\
I_{Z} = I_I + I_L \\
I_S = I_{Z_{min}} + I_{I_{max}} \\
I_S = I_{Z_{max}} + I_{I_{min}}
\]

**Timers**

- It is called timer IC
- It is used to generate pulse and square waveforms.
- The pin diagram of IC 555 is

\[
\begin{array}{cccc}
\text{Ground} & 1 & 8 & V_{CC} \\
\text{Trigger} & 2 & 5 & 7 \text{ Discharge} \\
\text{Output} & 3 & 5 & 6 \text{ Threshold} \\
\text{Reset} & 4 & 5 & 5 \text{ Control}
\end{array}
\]
The internal block diagram is

- The control input pin is used to change the voltage appearing at inverting node of upper comparator. Otherwise at that node zero voltage appears.
- The Reset pin is always connected to $V_{CC}$ otherwise reset transistor is ON and hence output is zero.

**Application of 555.**

**Monostable multivibrator:**

$T_0 = 1.1RC$

**Astable Multivibrator:**

$T_{High} = 0.69(R_A + R_B)C$ charging time constant.

$T_{Low} = 0.69R_B C$ discharging time constant.

$T_0 = 0.69(R_A + 2R_B)C$

$\frac{1.45}{(R_A + 2R_B)C}$
Duty cycle, \( D = \frac{R_A + R_B}{R_A + 2R_B} \)

**Voltage controlled Oscillator:**

- A voltage controlled oscillator (VCO) is an electronic oscillator whose oscillation frequency is controlled by a voltage input.
- The applied input voltage determined the instantaneous oscillation frequency.
1. **Binary to Decimal Conversion:**

\[(101.01)_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-1}\]

\[= 1 \times 2^2 + 2^0 + 2^{-1} + 2^{-1} = 5.25_{10}\]

2. **Decimal to Binary Conversion:**

A double dabble method is used for decimal to binary conversion. It is explained by following example.

Ex: Convert decimal number 23.375 to its binary equivalent.

**Integer part:**

\[
\begin{array}{c|c|c}
2 & 23 & 1 \\
2 & 12 & 0 \\
2 & 6  & 0 \\
2 & 3  & 1 \\
1 &   & \\
\end{array}
\]

\[\therefore (23)_{10} = (11001)_{2}\]

**Fractional part:**

\[
\begin{array}{c|c|c}
0.375 \times 2 = 0.75 & \rightarrow 0 \\
0.75 \times 2 = 1.5 & \rightarrow 1 \\
0.5 \times 2 = 1.0 & \rightarrow 1 \\
0.0 \times 2 = 0 & \rightarrow 0 \\
\end{array}
\]

Carriers read down top to bottom.

\[\therefore (0.375)_{10} = (.011)_{2}\]

\[\therefore (23.375)_{10} = (11001.011)_{2}\]

**Binary Arithmetic:**

<table>
<thead>
<tr>
<th>Binary Addition</th>
<th>Binary Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 + 0 = 0)</td>
<td>(0 - 0 = 0)</td>
</tr>
<tr>
<td>(0 + 1 = 1)</td>
<td>(1 - 0 = 1)</td>
</tr>
<tr>
<td>(1 + 0 = 1)</td>
<td>(1 - 1 = 0)</td>
</tr>
<tr>
<td>(1 + 1 = 10)</td>
<td>(10 - 1 = 1)</td>
</tr>
</tbody>
</table>

\[1 + 1 + 1 = 11\]

**Signed Numbers:**

- **Sign Magnitude**
  1. MSB used to represent sign (0 for +ve and 1 for –ve).
  2. Remaining bits used to represent magnitude of the number.
  3. Range of signed magnitude number is \(-\left(2^{n-1} - 1\right)\) to \(+\left(2^{n-1} - 1\right)\).

- **One’s Complement Notation**
  1. In a binary number, If we replace each 0 by 1 and each 1 by 0, we obtain another binary number which is one’s complement of the first number.
  2. MSB used to represent sign (0 for +ve and 1 for –ve).
  3. If MSB=0, just take remaining bits as it is, to find magnitude.
4. If MSB=1, take remaining bits as one’s complement form, to obtain magnitude of number.
5. Range: $-(2^{n-1} - 1) + (2^{n-1} - 1)$

- **Two's Complement Notation**
  1. By adding 1 to number we get the one’s complement of a binary number we get the two’s complement of that binary number.
  2. MSB used to represent sign (0 for +ve and 1 for –ve).
  3. If MSB=0, just take remaining bits as it is, to find magnitude.
  4. If MSB=1, take 2’s complement to remaining bits, to obtain magnitude of number.
  5. Range: $-2^{n-1} + (2^{n-1} - 1)$

- **Octal Number System:**
  Sets of 3-bit binary numbers can be conveniently represented by octal numbers with base 8. These numbers are 0, 1,2,3,4,5,6,7.
  
  **Note:** Conversion of Decimal number to octal number is same as decimal to binary. But here base is 8.

- **Octal to Binary Conversion:**
  Octal numbers can be converted to equivalent binary numbers by replacing each digit by its 3-bit binary equivalent.

- **Binary to Octal Conversion:**
  - For integer part, binary numbers can be converted into equivalent octal numbers by making groups of 3-bits starting from LSB to MSB.
  - For fractional part, we start grouping from the bit next to binary point and move towards right.

- **Hexadecimal Number System:**
  Sets of 4-bit binary numbers can be conveniently represented by Hex numbers with base 16. These numbers are 0, 1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.
  
  **Note:** Conversion of Decimal number to Hex number is same as decimal to binary. But here base is 16.

- **Hexadecimal to Binary Conversion:**
  Hex numbers can be converted to equivalent binary numbers by replacing each digit by its 4-bit binary equivalent.

- **Binary to Hex Conversion:**
  - For integer part, binary numbers can be converted into equivalent Hex numbers by making groups of 4-bits starting from LSB to MSB.
  - For fractional part, we start grouping from the bit next to binary point and move towards right.

- **Codes:**
  - **BCD Code:**
    - Binary coded decimal
- Weighted code
- 4 bit code
- 8421 code

- Excess-3 Code:
  - Excess-3 code = BCD + 3
  - Unweighted code
  - 4 bit code
  - It is self-compliment code

- Binary to Gray:
  - Unweighted code
  - Successive code differs by 1 bit.
  - Unit distance code.

  \[
  \begin{array}{ccccc}
  b(1) & b(2) & b(3) & b(4) & b(5) \\
  1 & 0 & 0 & 1 & 1 \\
  g(1) & g(2) & g(3) & g(4) & g(5) \\
  1 & 0 & 0 & 1 & 1 \\
  \end{array}
  \]

- Gray to Binary:

  \[
  \begin{array}{ccccc}
  g(1) & g(2) & g(3) & g(4) & g(5) \\
  1 & 0 & 0 & 1 & 1 \\
  b(1) & b(2) & b(3) & b(4) & b(5) \\
  1 & 1 & 1 & 0 & 1 \\
  \end{array}
  \]

- NOT Gate:

  \[
  A \rightarrow \overline{A}
  \]

  \[
  \begin{array}{|c|c|}
  \hline
  \text{Input (A)} & \text{Output (A)} \\
  \hline
  0 & 1 \\
  1 & 0 \\
  \hline
  \end{array}
  \]

- OR Gate:

  \[
  A + B
  \]

  \[
  \begin{array}{|c|c|c|}
  \hline
  \text{Input(A)} & \text{Input(B)} & \text{Output(A + B)} \\
  \hline
  0 & 0 & 0 \\
  0 & 1 & 1 \\
  1 & 0 & 1 \\
  1 & 1 & 1 \\
  \hline
  \end{array}
  \]

- AND Gate:

  \[
  A \cdot B
  \]

  \[
  \begin{array}{|c|c|c|}
  \hline
  \text{Input(A)} & \text{Input(B)} & \text{Output(A . B)} \\
  \hline
  0 & 0 & 0 \\
  0 & 1 & 0 \\
  1 & 0 & 0 \\
  1 & 1 & 1 \\
  \hline
  \end{array}
  \]

- NOR Gate:

  \[
  \overline{A + B}
  \]

  \[
  \begin{array}{|c|c|c|}
  \hline
  \text{Input(A)} & \text{Input(B)} & \text{Output(\overline{A + B})} \\
  \hline
  0 & 0 & 1 \\
  0 & 1 & 0 \\
  1 & 0 & 0 \\
  1 & 1 & 0 \\
  \hline
  \end{array}
  \]
NAND Gate:

\[
\begin{array}{ccc}
\text{Input(A)} & \text{Input(B)} & \text{Output(\overline{A}. \overline{B})} \\
0 & 0 & 1 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0 \\
\end{array}
\]

XOR Gate:

\[
\begin{array}{ccc}
\text{Input(A)} & \text{Input(B)} & \text{Output(\overline{A}B + AB)} \\
0 & 0 & 0 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0 \\
\end{array}
\]

XNOR Gate:

\[
\begin{array}{ccc}
\text{Input(A)} & \text{Input(B)} & \text{Output(\overline{A}B + \overline{A}B)} \\
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
1 & 1 & 1 \\
\end{array}
\]

Boolean algebra:

<table>
<thead>
<tr>
<th>OR</th>
<th>AND</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x + 0 = x)</td>
<td>(x . 0 = 0)</td>
</tr>
<tr>
<td>(x + x = x)</td>
<td>(x . x = x)</td>
</tr>
<tr>
<td>(x + 1 = 1)</td>
<td>(x . 1 = x)</td>
</tr>
<tr>
<td>(x + \overline{x} = 1)</td>
<td>(x . \overline{x} = 0)</td>
</tr>
</tbody>
</table>

Commutation: \(x + y = y + x\)

\(x . y = y . x\)

Association: \((x + y) + z = x + (y + z)\)

\((x . y) . z = x . (y . z)\)

Distribution: \(x . (y + z) = x . y + x . z\)

\(x + y . z = (x + y)(x + z)\)

DeMorgan’s Theorem:

1. \(\overline{x + y} = \overline{x} \cdot \overline{y}\)
2. \(x \cdot y = \overline{x + \overline{y}}\)

Duality Theorem:

1. Change each OR sign to AND sign and vice-versa.
2. Complementing all 0’s and 1’s.

Sum of Products (SOP) / Products of Sums (POS):

1. Any Boolean function implied by a truth table may be stated as SOP or POS.
2. Here each term contains all the inputs.
3. This is the canonical or standard form.
4. Each term in standard SOP is called \textit{min} term.
5. Each term in standard POS is called \textit{max} term.
6. Two logically different functions don’t contain same set of min terms (max terms).
7. For \(n\)-inputs there are \(2^n\) min terms (max terms).

Example: 3-input EXOR

SOP Form:

\[f(x, y, z) = \sum m(1, 2, 4, 7) \rightarrow \text{Minterms}\]

\[f(x, y, z) = \overline{x}yz + \overline{x}yz + xy\overline{z} + xyz\]
Minimization of Boolean Functions:
Simplifying the Boolean function will enable the number of gates required to be reduced.

1. Algebraic manipulation (as seen above)
2. Karnaugh (K) mapping (a visual approach)
3. Tabular approaches (usually implemented by computer, e.g., Quine-McCluskey)

K-map: It is the preferred technique for up to about 5 variables.

1. The required boolean results are transferred from a truth table onto a two-dimensional grid where the cells are ordered in Gray code.
2. Solution need not be unique.

#### Combinational Circuits

- **Half Adder:**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>S</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

  \[S = AB + A \bar{B} = A \oplus B\]

- **Full Adder:**

  \[
  \begin{array}{cccccc}
  A & B & C_i & S & C_o \\
  \hline
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 1 & 0 \\
  0 & 1 & 0 & 1 & 0 \\
  0 & 1 & 1 & 0 & 1 \\
  1 & 0 & 0 & 1 & 0 \\
  1 & 0 & 1 & 0 & 1 \\
  1 & 1 & 0 & 0 & 1 \\
  1 & 1 & 1 & 1 & 1 \\
  \end{array}
  \]

  \[Sum: S(A,B,C_i) = \sum m(1,2,4,7)\]
  \[Carry: C_o(A,B,C_i) = \sum m(3,5,6,7)\]

- **Decoder:**

  N inputs, \(2^N\) outputs.

  2 – to – 4 – line decoder with enable (E) input.
Encoder:

- An encoder is a digital circuit that performs the inverse operation of a decoder.
- An encoder has $2^N$ input lines and $N$ output lines.
- Truth Table of octal – to – Binary Encoder.

<table>
<thead>
<tr>
<th>E</th>
<th>A</th>
<th>B</th>
<th>D₀</th>
<th>D₁</th>
<th>D₂</th>
<th>D₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>x</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

De multiplexers:

- A de-multiplexer is a circuit that receives information on a single line and transmits this information on one of $2^N$ possible output lines.
- The selection of a specific output line is controlled by the bit values of $n$ selection lines. A decoder with an enable input is referred to as a decoder / de multiplexer. It is the enable input that makes the circuit a de multiplexer.

Multiplexers:

- A digital multiplexer is a combinational circuit that selects binary information from one of many input lines and directs it to a single output line. Normally there are $2^N$ input lines and $N$ selection lines whose bit combinations determine which input is selected.
A multiplexer is also called a data selector, since it selects one of many inputs and steers the binary information to the output line.

**SR Flip-Flop:**

<table>
<thead>
<tr>
<th>S</th>
<th>R</th>
<th>( Q_{n+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>( Q_n )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>Invalid</td>
</tr>
</tbody>
</table>

**J-K Flip-Flop:**

<table>
<thead>
<tr>
<th>J</th>
<th>K</th>
<th>( Q_{n+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>( Q_n )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>( \overline{Q}_n )</td>
</tr>
</tbody>
</table>

J-K flip-flop is also known as universal flip-flop.

**D Flip-Flop:**

<table>
<thead>
<tr>
<th>D</th>
<th>( Q_{n+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**T Flip-Flop:**

<table>
<thead>
<tr>
<th>T</th>
<th>( Q_{n+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( Q_n )</td>
</tr>
<tr>
<td>1</td>
<td>( \overline{Q}_n )</td>
</tr>
</tbody>
</table>

**Level Trigger:** In level triggering output can change number of times in single clock.

There are two types of level triggering techniques

1. Positive Level Triggering
2. Negative Level Triggering.

**Edge Triggering:** In edge triggering output changes only at transition of clock signal.
There are two types of edge triggering techniques.
1. Positive edge triggering
2. Negative edge triggering

**Race around condition:**
Occurs in JK FF if J=K=1 or
Propagation delay of FF is less than Clock Pulse width \( t_{pd} < t_{pw} \)
Condition to avoid Race around Condition:
\( t_{pw} < t_{pd} < T \) or By using Master Slave FF

**Counters:**
With N flip flops, maximum possible states in counter are \( 2^N \).

**Asynchronous Counter:**
- Different Clock Pulse.
- Slower in operation.
- Fixed Count sequence(up/down)

**Mod N Counter:**
\[
Output Frequency = \frac{Input Frequency}{N}
\]
If mod M and mod N counter are cascaded then resultant counter is mod MN counter.

**Ripple Counter:**
- Asynchronous Counter.
- FF are connected in toggling mode.
- Output of one FF act as clock to next FF.
- Input is applied to 1st FF i.e LSB FF.

**Synchronous Counter:**
Single Clock will be shared among all FF.

- **Ring Counter:**
  - SISO counter.
  - Output of last FF is connected to input of first FF.
  - N FF, N different states.
  - No of un used states \( =2^N - N \)
  - \( Output \ Frequency = \frac{Input \ Frequency}{N} \)

- **Johnson Counter:**
  - Inverted output of last FF is connected to input of first FF.
  - N FF, 2N different states.
  - No of un used states \( =2^N - 2N \)
  - \( Output \ Frequency = \frac{Input \ Frequency}{2N} \)

- **Registers:**
  Holds a group of bits
To store n bits, n FF are required.
PIPO is not exactly shift register.

<table>
<thead>
<tr>
<th></th>
<th>Clock Pulses Required to store n bits</th>
<th>Clock Pulses Required to read n bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>SISO</td>
<td>n</td>
<td>n-1</td>
</tr>
<tr>
<td>SIPO</td>
<td>n</td>
<td>0</td>
</tr>
<tr>
<td>PISO</td>
<td>1</td>
<td>n-1</td>
</tr>
<tr>
<td>PIPO</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Left shift by one bit: multiply by 2
Right shift by one bit: divide by 2
Logic Gate Families

- **Diode Logic (DL):**
  AND logic with Diode Logic:
  
  $V_{CL} = 5V$

  ![Diagram of Diode Logic](image)

  - Simplest; does not scale.
  - NOT logic is not possible (need an active element).

- **Resistor-Transistor Logic (RTL):**

  ![Diagram of RTL](image)

  - Replace diode switch with a transistor switch
  - Can be cascaded
  - Large power draw

- **Diode-Transistor Logic (DTL):**

  ![Diagram of DTL](image)

  - Essentially diode logic with transistor amplification
  - Reduced power consumption
  - Faster than RTL

- **Transistor-Transistor Logic (TTL):**

  ![Diagram of TTL](image)

  - A major slowdown factor in BJTs is due to transistors going in/out of saturation
  - Schottky diode has a lower forward bias (0.25V)
  - When BC junction would become forward biased, the Schottky diode

bypasses the current preventing the transistor from going into saturation.

<table>
<thead>
<tr>
<th>Emitter-Coupled Logic (ECL):</th>
<th>Very low static power consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PROS:</strong> Fastest logic family available (~1ns)</td>
<td>Scaling capabilities (large integration all MOS)</td>
</tr>
<tr>
<td><strong>CONS:</strong> Low noise margin and high power dissipation</td>
<td>TTL power essentially constant (no frequency dependence)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Complimentary MOS (CMOS):</th>
<th>CMOS power scales as $\alpha f \times C \times V^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Other variants:</strong> NMOS, PMOS (obsolete)</td>
<td>At high frequencies (&gt;&gt; MHz) CMOS dissipates more power than TTL</td>
</tr>
<tr>
<td></td>
<td>Overall advantage is still for CMOS even for very fast chips.</td>
</tr>
</tbody>
</table>
### Microprocessors

#### Features:
1. 8-bit microprocessor
2. 8-data lines and 16-address lines
3. Addressing capacity =$2^{16} = 64KB$
4. Clock frequency $3MHz$
5. Requires +5V power supply
6. It is a single chip NMOS device, contains 6200 transistors
7. It provides 74 instructions with 5 addressing modes.
8. It provides 5 hardware and 8 software interrupts.

#### Pin Configuration:
1. **Address Bus:** (A15-A8 and AD7-AD0)
2. **Data Bus:** (AD7-AD0) are often called as multiplexed data lines.
3. **Control Lines:**
   - $RD$: Read
   - $WR$: Write
   - $IO / M$: input-output/memory
4. **Status Lines:**

<table>
<thead>
<tr>
<th>$S_1$</th>
<th>$S_0$</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>HLT state</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>Writing operation</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>Reading Operation</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>Op code fetch operation</td>
</tr>
</tbody>
</table>

5. **Special Signal:**
   - ALE: Address Latch Enable: used to enable or disable the external IC
   - External IC is used for de-multiplexing of AD7-AD0 lines. i.e. It is used to separate the address and data from AD7-AD0 lines.
   - If ALE=1/0 → IC enabled/ disabled

#### Registers:
- A - Accumulator – it is a special purpose register. All the ALU operations are performed with reference to the content of Accumulator.
- BCDEHL- General purpose registers. These registers also used for 16 bit operations in pairs. The default pairs are BC, DE and HL.
- F-Flag register: It indicate status of ALU operation.
- PC: Program Counter: this is a 16 bit register, used to address the memory location where next instruction is going to be executed.
- SP-Stack Pointer: it is 16 –bit register, holds the address of top of stack
- Temporary registers (W,Z): these used only by processor, not available for programmer.
### Addressing modes:

- **Immediate:** (MOV A,B; ADD B; ANA C)
- **Register:** (MVI A,05H; LXI B,2049H; ORI 07H)
- **Direct:** (LDA 5024H; STA 5900H; IN 09H; OUT 70H)
- **Indirect:** (MOV A,M; MOV M,A; ADD M; ORA M)
- **Implied (Implicit):** (HLT; NOP; RST; RET)

### Instruction Set:

#### Data Transfer Instructions:

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Source</th>
<th>Destination</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOV</td>
<td>Rd, Rs</td>
<td>M, Rs</td>
<td>Copy the contents of the source register into the destination register; If one of the operands is a memory location, its location is specified by the contents of the HL registers. <strong>Example:</strong> MOV B, C or MOV B, M</td>
</tr>
<tr>
<td>MVI</td>
<td>Rd, data</td>
<td>M, data</td>
<td>The 8-bit data is stored in the destination register or memory. If the operand is a memory location, its location is specified by the contents of the HL registers. <strong>Example:</strong> MVI B, 57H or MVI M, 57H</td>
</tr>
<tr>
<td>LDA</td>
<td>16-bit address</td>
<td></td>
<td>The contents of a memory location, specified by a 16-bit address in the operand, are copied to the accumulator. <strong>Example:</strong> LDA 2034H or LDA XYZ</td>
</tr>
<tr>
<td>LDAX</td>
<td>B/D Reg. pair</td>
<td></td>
<td>This instruction copies the contents of that memory location into the accumulator. <strong>Example:</strong> LDAX B</td>
</tr>
<tr>
<td>LXI</td>
<td>Reg. pair, 16-bit data</td>
<td></td>
<td>The instruction loads 16-bit data in the register pair designated in the operand. <strong>Example:</strong> LXI H, 2034H</td>
</tr>
<tr>
<td>Instruction</td>
<td>Operand Type</td>
<td>Description</td>
<td>Example</td>
</tr>
<tr>
<td>---------------</td>
<td>----------------</td>
<td>-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>LHLD</td>
<td>16-bit address</td>
<td>The instruction copies the contents of the memory location pointed out by the 16-bit address into register L and copies the contents of the next memory location into register H. The contents of source memory locations are not altered. Example: LHLD 2040H</td>
<td></td>
</tr>
<tr>
<td>STA</td>
<td>16-bit address</td>
<td>The contents of the accumulator are copied into the memory location specified by the operand. This is a 3-byte instruction, the second byte specifies the low-order address and the third byte specifies the high-order address. Example: STA 4350 or STA XYZ</td>
<td></td>
</tr>
<tr>
<td>STAX</td>
<td>Reg. pair</td>
<td>The contents of the accumulator are copied into the memory location specified by the contents of the operand (register pair). The contents of the accumulator are not altered. Example: STAX B</td>
<td></td>
</tr>
<tr>
<td>SHLD</td>
<td>16-bit address</td>
<td>The contents of register L are stored into the memory location specified by the 16-bit address in the operand and the contents of H register are stored into the next memory location by incrementing the operand. The contents of registers HL are not altered. This is a 3-byte instruction, the second byte specifies the low-order address and the third byte specifies the high-order address. Example: SHLD 2470</td>
<td></td>
</tr>
<tr>
<td>XCHG</td>
<td>None</td>
<td>Exchange H and L with D and E Example: XCHG</td>
<td></td>
</tr>
<tr>
<td>SPHL</td>
<td>None</td>
<td>Copy H and L registers to the stack pointer Example: SPHL</td>
<td></td>
</tr>
<tr>
<td>XTHL</td>
<td>None</td>
<td>Exchange H and L with top of stack</td>
<td></td>
</tr>
</tbody>
</table>
### PUSH
(Push register pair onto stack)

| Reg. pair | The stack pointer register is decremented and the contents of the high order register (B, D, H, A) are copied into that location. The stack pointer register is decremented again and the contents of the low-order register (C, E, L, flags) are copied to that location. 
**Example:** PUSH B or PUSH A |

### POP
(Pop off stack to register pair)

| Reg. pair | The contents of the memory location pointed out by the stack pointer register are copied to the low-order register (C, E, L, status flags) of the operand. The stack pointer is incremented by 1 and the contents of that memory location are copied to the high-order register (B, D, H, A) of the operand. The stack pointer register is again incremented by 1. 
**Example:** POP H or POP A |

### OUT
8-bit port address

| Output data from accumulator to a port with 8-bit address |
| **Example:** OUT 87 |

### IN
8-bit port address

| Input data to accumulator from a port with 8-bit address |

### Arithmetic Instructions:

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADD R M</td>
<td>Add register or memory to accumulator. All flags are modified to reflect the result of the addition.</td>
<td></td>
</tr>
<tr>
<td><strong>Example:</strong> ADD B or ADD M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADC R M</td>
<td>Add register to accumulator with carry. All flags are modified to reflect the result of the addition.</td>
<td></td>
</tr>
<tr>
<td><strong>Example:</strong> ADC B or ADC M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADI 8-bit data</td>
<td>Add immediate to accumulator. All flags are modified to reflect the result of the addition.</td>
<td></td>
</tr>
<tr>
<td><strong>Example:</strong> ADI 45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACI 8-bit data</td>
<td>Add immediate to accumulator with carry. All flags are modified to reflect the result of the addition.</td>
<td></td>
</tr>
<tr>
<td><strong>Example:</strong> ACI 45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Instruction</td>
<td>Reg/Pair</td>
<td>Description</td>
</tr>
<tr>
<td>-------------</td>
<td>----------</td>
<td>-------------</td>
</tr>
<tr>
<td>DAD</td>
<td>Reg.Pair</td>
<td>Add register pair to H and L registers. If the result is larger than 16 bits, the CY flag is set. No other flags are affected.</td>
</tr>
<tr>
<td>SUB</td>
<td>R</td>
<td>Subtract register or memory from accumulator. All flags are modified to reflect the result of the subtraction.</td>
</tr>
<tr>
<td>SBB</td>
<td>R</td>
<td>Subtract source and borrow from accumulator. All flags are modified to reflect the result of the subtraction.</td>
</tr>
<tr>
<td>SUI</td>
<td>8-bit data</td>
<td>Subtract immediate from accumulator. All flags are modified to reflect the result of the subtraction.</td>
</tr>
<tr>
<td>SBI</td>
<td>8-bit data</td>
<td>Subtract immediate from accumulator with borrow. All flags are modified to reflect the result of the subtraction.</td>
</tr>
<tr>
<td>INR</td>
<td>R</td>
<td>Increment register or memory by 1. All flags effect except CY flag.</td>
</tr>
<tr>
<td>INX</td>
<td>R</td>
<td>Increment register pair by 1. No flags are affected.</td>
</tr>
<tr>
<td>DCR</td>
<td>R</td>
<td>Decrement register or memory by 1. All flags effect except CY flag.</td>
</tr>
<tr>
<td>DCX</td>
<td>R</td>
<td>Decrement register pair by 1. No flags are affected.</td>
</tr>
<tr>
<td>DAA</td>
<td>None</td>
<td>Decimal adjust accumulator. The contents of the accumulator are changed from a binary value to two 4-bit binary coded decimal (BCD) digits. This is the only</td>
</tr>
</tbody>
</table>
instruction that uses the auxiliary flag to perform the binary to BCD conversion, and the conversion procedure is described below. S, Z, AC, P, CY flags are altered to reflect the results of the operation. If the value of the low-order 4-bits in the accumulator is greater than 9 or if AC flag is set, the instruction adds 6 to the low-order four bits. If the value of the high-order 4-bits in the accumulator is greater than 9 or if the Carry flag is set, the instruction adds 6 to the high-order four bits.

**Example:** DAA

### Branching instructions:

<table>
<thead>
<tr>
<th>JMP (Jump unconditionally)</th>
<th>16-bit address</th>
<th><strong>Example:</strong> JMP 2034 or JMP XYZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jump conditionally</td>
<td>16-bit address</td>
<td><strong>Example:</strong> JZ 2034 or JZ XYZ</td>
</tr>
<tr>
<td></td>
<td></td>
<td>JC Jump on Carry CY = 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>JNC Jump on no Carry CY = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>JP Jump on positive S = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>JM Jump on minus S = 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>JZ Jump on zero Z = 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>JNZ Jump on no zero Z = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>JPE Jump on parity even P = 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>JPO Jump on parity odd P = 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CALL (Unconditional subroutine call)</th>
<th>16-bit address</th>
<th>The program sequence is transferred to the memory location specified by the 16-bit address given in the operand. Before the transfer, the address of the next instruction after CALL (the contents of the program counter) is pushed onto the stack. <strong>Example:</strong> CALL 2034 or CALL XYZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call conditionally</td>
<td>16-bit address</td>
<td><strong>Example:</strong> CZ 2034 or CZ XYZ</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CC Call on Carry CY = 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CNC Call on no Carry CY = 0</td>
</tr>
<tr>
<td>Instruction</td>
<td>Source Code</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------</td>
<td>-------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>CP</td>
<td>Call on positive $S = 0$</td>
<td></td>
</tr>
<tr>
<td>CM</td>
<td>Call on minus $S = 1$</td>
<td></td>
</tr>
<tr>
<td>CZ</td>
<td>Call on zero $Z = 1$</td>
<td></td>
</tr>
<tr>
<td>CNZ</td>
<td>Call on no zero $Z = 0$</td>
<td></td>
</tr>
<tr>
<td>CPE</td>
<td>Call on parity even $P = 1$</td>
<td></td>
</tr>
<tr>
<td>CPO</td>
<td>Call on parity odd $P = 0$</td>
<td></td>
</tr>
<tr>
<td>RET (Return from subroutine unconditionally)</td>
<td>none</td>
<td>The program sequence is transferred from the subroutine to the calling program. The two bytes from the top of the stack are copied into the program counter, and program execution begins at the new address. <strong>Example:</strong> RET</td>
</tr>
</tbody>
</table>
| Return from subroutine conditionally | none | **Example:** RZ  
  RC Return on Carry $CY = 1$  
  RNC Return on no Carry $CY = 0$  
  RP Return on positive $S = 0$  
  RM Return on minus $S = 1$  
  RZ Return on zero $Z = 1$  
  RNZ Return on no zero $Z = 0$  
  RPE Return on parity even $P = 1$  
  RPO Return on parity odd $P = 0$ |
| PCHL              | none | Load program counter with HL contents  **Example:** PCHL |
| RST (Restart)     | 0-7 | Instruction Restart Address  
  RST 0 0000H  
  RST 1 0008H  
  RST 2 0010H  
  RST 3 0018H  
  RST 4 0020H  
  RST 5 0028H  
  RST 6 0030H  
  RST 7 0038H |
Logical Instructions:

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
</table>
| **CMP**     | Compare register or memory with accumulator | if (A) < (reg/mem): carry flag is set, s=1  
if (A) = (reg/mem): zero flag is set, s=0  
if (A) > (reg/mem): carry and zero flags are reset, s=0  
**Example:** CMP B or CMP M |
| **CPI**     | Compare immediate with accumulator | if (A) < data: carry flag is set, s=1  
if (A) = data: zero flag is set, s=0  
if (A) > data: carry and zero flags are reset, s=0  
**Example:** CPI 89 |
| **ANA**     | Logical AND register or memory with accumulator | S, Z, P are modified to reflect the result of the operation. CY is reset. AC is set.  
**Example:** ANA B or ANA M |
| **ANI**     | Logical AND immediate with accumulator | S, Z, P are modified to reflect the result of the operation. CY is reset. AC is set.  
**Example:** ANI 86 |
| **XRA**     | Exclusive OR register or memory with accumulator | S, Z, P are modified to reflect the result of the operation. CY and AC are reset.  
**Example:** XRA B or XRA M |
| **XRI**     | Exclusive OR immediate with accumulator | S, Z, P are modified to reflect the result of the operation. CY and AC are reset.  
**Example:** XRI 8-bit data  
<p>|</p>
<table>
<thead>
<tr>
<th>Instruction</th>
<th>Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC are reset.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Example:</strong> XRI 86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ORA</td>
<td>R M</td>
<td>Logical OR register or memory with accumulator S, Z, P are modified to reflect the result of the operation. CY and AC are reset. <strong>Example:</strong> ORA B or ORA M</td>
</tr>
<tr>
<td>ORI</td>
<td>8-bit data</td>
<td>Logical OR immediate with accumulator S, Z, P are modified to reflect the result of the operation. CY and AC are reset. <strong>Example:</strong> ORI 86</td>
</tr>
<tr>
<td>RLC</td>
<td>none</td>
<td>Rotate accumulator left Only CY flag affects( CY affects according to D7) <strong>Ex:</strong> RLC</td>
</tr>
<tr>
<td>RRC</td>
<td>none</td>
<td>Rotate accumulator right Only CY flag affects( CY affects according to D0) <strong>Ex:</strong> RRC</td>
</tr>
<tr>
<td>RAL</td>
<td>none</td>
<td>Rotate accumulator left through carry Only CY flag affects( CY affects according to D7) <strong>Ex:</strong> RAL</td>
</tr>
<tr>
<td>RAR</td>
<td>none</td>
<td>Rotate accumulator right through carry Only CY flag affects( CY affects according to D0) <strong>Ex:</strong> RAL</td>
</tr>
<tr>
<td>CMA</td>
<td>none</td>
<td>Complement accumulator, Example: CMA No flags are affected.</td>
</tr>
<tr>
<td>CMC</td>
<td>none</td>
<td>Complement carry, Example: CMC The Carry flag is complemented. No other flags are affected</td>
</tr>
<tr>
<td>STC</td>
<td>none</td>
<td>Set Carry The Carry flag is set to 1. No other flags are affected. <strong>Example:</strong> STC</td>
</tr>
</tbody>
</table>
## Control Instructions:

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Flags</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOP</td>
<td>none</td>
<td>No operation</td>
</tr>
<tr>
<td>HLT</td>
<td>None</td>
<td>Halt and enter wait state</td>
</tr>
<tr>
<td>DI</td>
<td>None</td>
<td>Disable interrupts, No flags are affected.</td>
</tr>
<tr>
<td>EI</td>
<td>None</td>
<td>Enable interrupts, No flags are affected.</td>
</tr>
</tbody>
</table>
| RIM (Read interrupt mask) | None | **Example: RIM**  
This is a multipurpose instruction used to read the status of interrupts 7.5, 6.5, 5.5 and read serial data input bit. The instruction loads eight bits in the accumulator with the following interpretations. |
| SIM (Set interrupt mask) | None | **Example: SIM**  
This is a multipurpose instruction and used to implement the 8085 interrupts 7.5, 6.5, 5.5, and serial data output. The instruction interprets the accumulator contents as follows. |

![Diagram](https://via.placeholder.com/150)
SOD — Serial Output Data: Bit $D_7$ of the accumulator is latched into the SOD output line and made available to a serial peripheral if bit $D_0 = 1$.

SDE — Serial Data Enable: If this bit = 1, it enables the serial output. To implement serial output, this bit needs to be enabled.

XXX — Don’t Care

R7.5 — Reset RST 7.5: If this bit = 1, RST 7.5 flip-flop is reset. This is an additional control to reset RST 7.5.

MSE — Mask Set Enable: If this bit is high, it enables the functions of bits $D_2$, $D_1$, $D_0$. This is a master control over all the interrupt masking bits. If this bit is low, bits $D_2$, $D_1$, and $D_0$ do not have any effect on the masks.

$M7.5 - D_2 = 0$, RST 7.5 is enabled.
$= 1$, RST 7.5 is masked or disabled.

$M6.5 - D_1 = 0$, RST 6.5 is enabled.
$= 1$, RST 6.5 is masked or disabled.

$M5.5 - D_0 = 0$, RST 5.5 is enabled.
$= 1$, RST 5.5 is masked or disabled.
Signals and Systems

Standard Signals

- **Unit Step:** \( u(t) \)
  \[
  u(t) = \begin{cases} 
  1; & t > 0 \\
  0; & t < 0 \\
  u(0) = \frac{1}{2}
  \end{cases}
  \]

- **Rectangular or Gate Function**
  \[
  x(t) = \begin{cases} 
  A; & -\frac{T}{2} < t < \frac{T}{2} \\
  0; & \text{else where}
  \end{cases}
  \]
  \[
  x(t) = A \text{rect}\left(\frac{t}{T}\right) \text{ or } A\pi\left(\frac{t}{T}\right)
  \]

- **Continuous Time Impulse or Derac Delta Function:** \( \delta(t) \)
  \[
  \delta(t) = \begin{cases} 
  \infty; & t = 0 \\
  0; & t \neq 0
  \end{cases}
  \]
  \[
  \delta(t) = \lim_{T \to 0} \frac{1}{T} \text{rect}\left(\frac{t}{T}\right) = \frac{du(t)}{dt}
  \]

- **Properties of Unit Impulse**
  a) \( \int_{-\infty}^{\infty} \delta(t) \, dt = 1 \)
  b) \( \delta(-t) = \delta(t) \rightarrow \) even function
  c) \( \delta(at) = \frac{1}{a} \delta(t) \rightarrow \) Scaling
  d) \( x(t)\delta(t - to) = x(to)\delta(t - to) \)
  e) \( x(t) \ast \delta(t - to) = x(t - to) \)
  f) \( \int_{t_1}^{t_2} x(t)\delta(t - to) \, dt = x(to); \quad t_1 < to < t_2 \)

- **Unit Ramp Function:** \( r(t) \)
  \[
  r(t) = tu(t) = \begin{cases} 
  t; & t > 0 \\
  0; & t \leq 0
  \end{cases}
  \]

- **Signum Function:** \( \text{sgn}(x) \)
  \[
  \text{sgn}(x) = \begin{cases} 
  1; & x > 0 \\
  0; & x = 0 \\
  -1; & x < 0
  \end{cases}
  \]

- **Sampling Function:**
  - In mathematics, the historical un-normalized \( \text{sinc} \) function is defined by,
    \[
    \text{Sinc}(x) = \frac{\sin x}{x}
    \]
  - In digital signal processing and information theory, the normalized \( \text{sinc} \) function is commonly defined by
    \[
    \text{Sinc}(x) = \frac{\sin \pi x}{\pi x}
    \]

- **Discrete Impulse Function:** \( \delta(n) \)
  \[
  \delta(n) = \begin{cases} 
  1; & n = 0 \\
  0; & \text{else}
  \end{cases}
  \]

Any obituary sequence can be generated by sum of scaled and delayed impulses.
\[
\delta(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)
\]

- **Properties:**
  1. \( x(n)\delta(n) = x(0) \)
  2. \( x(n)\delta(n-k) = x(k) \)

- **Unit step Sequence:** \( u(n) \)
  \[
  u(n) = \begin{cases} 
  1; & n \geq 0 \\
  0; & n < 0
  \end{cases}
  \]

- **Unit Ramp Sequence** \( r(n) \)
  \[
  r(n) = nu(n) = \begin{cases} 
  n; & n \geq 0 \\
  0; & n < 0
  \end{cases}
  \]

- **Relation between \( \delta(t), u(t) \) and \( r(t) \)**
  \[
  u(t) = \int_{-\infty}^{t} \delta(t) \, dt = \frac{dr(t)}{dx}
  \]

- **Relation between \( \delta(n), u(n) \) and \( r(n) \)**
  \[
  \delta(n) = u(n) - u(n-1)
  \]
  And
  \[
  u(n) = \sum_{m=0}^{\infty} \delta(n-m)\]
Classification of signals:

1. Energy and Power Signals

Continuous Time:
Energy of signal $x(t)$ is
$$E_x(t) = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt$$
Average Power of Signal $x(t)$ is $P_{avg} x(t) =$
$$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

Discrete Time:
Energy of signal $x(n)$ is
$$E_x(n) = \sum_{n=-\infty}^{\infty} |x(n)|^2$$
Average Power of Signal $x(n)$ is $P_{avg} x(n) =$
$$\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

2. Even and Odd Signals

- Even: $x(t) = x(-t)$ for Continuous time
  $x(n) = x(-n)$ for Discrete time
- Odd signal is symmetric about origin
- Odd: $x(t) = -x(-t)$ for Continuous Time
  $x(n) = -x(-n)$ for Discrete Time
- Odd signal passes through origin
- Every signal can be broken into even and odd parts.

CT domain:
$$x(t) = x_e(t) + x_o(t)$$
$$x(-t) = x_e(t) - x_o(t)$$
$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

3. Periodic and Non-periodic Signals

A signal is periodic if $(t) = x(t + T)$ , where $T$ is period of $x(t)$.
Where as in discrete domain
$$x(n) = x(n + N)$$ , where $N$ is period of $x(n)$.

Steps to find Period of sum of harmonic signals:

1. Identify individual time periods $T_1, T_2, T_3 \ldots \ldots$
2. Calculate $\frac{T_1}{T_2}, \frac{T_1}{T_3}, \ldots$ 
3. If the ratio of $2^{nd}$ step is rational number, then overall signal periodic.
4. Calculate the LCM of denominators of step 2.
5. Time Period $T = LCM * T_1$.

Finding the Time Period of discrete signals

Let $x(n) = sin\omega_0 n$

$$x(n + N) = sin \omega_0 (n + N)$$

$$= sin\omega_0 n cos\omega_0 N + cos\omega_0 n sin\omega_0 N$$

Signals to be periodic, $sin\omega_0 N = 0 = sin2\pi m$
\[ \omega_0 N = 2\pi m \]

\[ \frac{\omega_0}{2\pi} = \frac{m}{N} \]

Where \( N \) is period of signal.

- **Causal and Non-causal Signals**

  A signal is causal if
  
  \[ x(t) = 0; t < 0 \text{ for CT domain} \]
  \[ x(n) = 0; n < 0 \text{ for DT domain} \]

- **Classifications of systems**

  - **Linear systems:** A system which obeys the property of superposition is linear system. To check the linearity of system we need to examine the additive and scaling property.
  
  - **Time Invariant (Shift Invariant) & Time Variant (Shift dependent):** Time invariant system is one for which input and output characteristic doesn’t change with time.
  
  - **Causal (non-anticipative) and Non-Causal (anticipative) Systems:** A system is said to be causal if the present output depends only on present and past values of input but not on future values, otherwise non-causal system.
  
  - **Static (Memory less) system:** A system whose output doesn’t depend on future or past values and it depends only on present value.
  
  - **Dynamic (with Memory) system:** A system whose output depends on past and future values.

- **Stable and Unstable Systems (BIBO):**

  If a system is excited by bounded input (BI) and it produces bounded output (BO) then the system is Stable.

- **Invertible and Inverse Systems:** A system is invertible if the cascade of this system with its inverse system yields an output which is the input to the first system.

- **Convolution Sum:** For a given input \( x(t) \) and impulse response \( h(t) \) to a LTI system, then the output \( y(t) \) can be calculated by convolution sum of input and impulse response.

  \[
  y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \quad (or)
  \]

  \[
  y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \quad \rightarrow \text{CT domain}
  \]

  \[
  y(t) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)(or)
  \]

  \[
  y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) \quad \rightarrow \text{DT domain}
  \]

- **Trigonometric F.S.**

  \[
  g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t
  \]

  \[
  a_0 = \frac{1}{T} \int_{0}^{T} g(t)dt \rightarrow \text{DC (average value)}
  \]

  \[
  a_n = \frac{2}{T} \int_{0}^{T} g(t) \cos n\omega_0 t \ dt
  \]
\[ b_n = \frac{2}{T} \int_0^T g(t) \sin n\omega_0 t \, dt \]

- Polar form of T.F.S:
  \[ g(t) = d_0 + \sum_{n=1}^{\infty} d_n \cos(n\omega_0 t + \theta_n) \]
  \[ d_0 = a_0; \]
  \[ |d_n| = \sqrt{a_n^2 + b_n^2} \rightarrow \text{Magnitude Spectrum} \]
  \[ \theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right) \rightarrow \text{Phase Spectrum} \]

- Exponential F.S (or) Complex F.S:
  \[ g(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \]
  \[ c_n = \frac{1}{T} \int_0^T g(t) e^{-jn\omega_0 t} \, dt \]

- Convergence of F.S: (Dirichlet Conditions):
  1. \( x(t) \) must be absolutely integrable i.e.
     \[ \int_0^T |x(t)| \, dt < \infty \]
  2. \( x(t) \) has only a finite number of maxima and minima
  3. The no.of discontinuous in \( x(t) \) must be finite.

- Properties of the Continuous-Time Fourier Transform
  \[ x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} \, d\omega \]
  \[ X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt \]

- Properties of F.S:
  1. Linearity: \( x(t) \overset{FS}{\longleftrightarrow} c_n \) and \( y(t) \overset{FS}{\longleftrightarrow} d_n \) with period \( T \)
     Then \( ax(t) + \beta y(t) \overset{FS}{\longleftrightarrow} \alpha c_n + \beta d_n \)
  2. Time shifting: \( x(t) \overset{FS}{\longleftrightarrow} c_n \) then
     \( x(t - t_0) \overset{FS}{\longleftrightarrow} c_n e^{-jn\omega_0 t_0} \)
  3. Frequency shift: \( x(t) \overset{FS}{\longleftrightarrow} c_n \) then
     \( x(t) e^{j\omega_0 m} \overset{FS}{\longleftrightarrow} c_{n-m} \)
  4. Time scaling: \( x(t) \overset{FS}{\longleftrightarrow} c_n \) then
     \( x(\alpha t) \overset{FS}{\longleftrightarrow} c_n \)
     Time compressing by \( \alpha \) changes frequency from \( \omega_0 \) to \( \alpha \omega_0 \).
  5. Differentiation in time:
     \( x(t) \overset{FS}{\longleftrightarrow} c_n \) then
     \( \frac{dx(t)}{dt} \overset{FS}{\longleftrightarrow} (j\omega_0) c_n \)
  6. Parseval’s power theorem:
     \( x(t) \overset{FS}{\longleftrightarrow} c_n \) then
     \[ \frac{1}{T} \int_0^T |x(t)|^2 \, dt = \sum_{n=\infty}^{\infty} |c_n|^2 \]
<table>
<thead>
<tr>
<th>Property</th>
<th>Aperiodic Signal</th>
<th>Fourier Transform</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x(t) )</td>
<td>( X(j\omega) )</td>
</tr>
<tr>
<td></td>
<td>( y(t) )</td>
<td>( Y(j\omega) )</td>
</tr>
<tr>
<td>Linearity</td>
<td>( a x(t) + b y(t) )</td>
<td>( a X(j\omega) + b Y(j\omega) )</td>
</tr>
<tr>
<td>Time-shifting</td>
<td>( x(t - t_0) )</td>
<td>( e^{-j\omega t_0} X(j\omega) )</td>
</tr>
<tr>
<td>Frequency shifting</td>
<td>( e^{j\omega t} x(t) )</td>
<td>( X(j(\omega - \omega_0)) )</td>
</tr>
<tr>
<td>Conjugation</td>
<td>( x^*(t) )</td>
<td>( X^*(-j\omega) )</td>
</tr>
<tr>
<td>Time-Reversal</td>
<td>( x(-t) )</td>
<td>( X(-j\omega) )</td>
</tr>
<tr>
<td>Time and Frequency Scaling</td>
<td>( x(at) )</td>
<td>( \frac{1}{</td>
</tr>
<tr>
<td>Convolution</td>
<td>( x(t) * y(t) )</td>
<td>( X(j\omega)Y(j\omega) )</td>
</tr>
<tr>
<td>Multiplication</td>
<td>( x(t)y(t) )</td>
<td>( \frac{1}{2\pi} X(j\omega)*Y(j\omega) )</td>
</tr>
<tr>
<td>Differentiation in Time</td>
<td>( \frac{d}{dt} x(t) )</td>
<td>( j\omega X(j\omega) )</td>
</tr>
<tr>
<td>Integration</td>
<td>( \int_{-\infty}^{t} x(t)dt )</td>
<td>( \frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega) )</td>
</tr>
<tr>
<td>Differentiation in Frequency</td>
<td>( tx(t) )</td>
<td>( \frac{j}{d\omega} X(j\omega) )</td>
</tr>
</tbody>
</table>
| Conjugate Symmetry for Real Signals  | \( x(t) \) real  | \[X(j\omega) = X^*(-j\omega)\]  
\[\Re \{X(j\omega)\} = \Re \{X(-j\omega)\}\]  
\[\Im \{X(j\omega)\} = -\Im \{X(-j\omega)\}\]  
\[|X(j\omega)| = |X(-j\omega)|\]  
\[\angle X(j\omega) = -\angle X(-j\omega)\] |
| Symmetry for Real and Even Signals   | \( x(t) \) Real and even | \( X(j\omega) \) Real and Even |
| Symmetry for Real and Odd Signals    | \( x(t) \) Real and odd | \( X(j\omega) \) Purely imaginary and odd |
| Even-Odd Decomposition for Real Signals | \( x_e(t) = \text{Ev}\{x(t)\}\)  
\[\Re \{X(j\omega)\}\]  
\[j\Im \{X(j\omega)\}\] |
Parseval’s Relation for Aperiodic Signals,

\[
\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega
\]

**Basic Continuous-Time Fourier Transform Pairs**

<table>
<thead>
<tr>
<th>Signal</th>
<th>Fourier Transform</th>
<th>Fourier Series coefficients (if periodic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \sum_{k=-\infty}^{+\infty} a_k e^{j k \omega_0 t} ]</td>
<td>[ 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k \omega_0) ]</td>
<td>[ a_k ]</td>
</tr>
</tbody>
</table>
| \[ e^{j\omega_0 t} \] | \[ 2\pi \delta(\omega - \omega_0) \] | \[ a_1 = 1 \]
|  |  | \[ a_k = 0, \text{ otherwise} \] |
| \[ \cos \omega_0 t \] | \[ \pi \left[ \delta(\omega - \omega_0) + \delta(\omega + \omega_0) \right] \] | \[ a_1 = a_{-1} = \frac{1}{2} \]
|  |  | \[ a_k = 0, \text{ otherwise} \] |
| \[ \sin \omega_0 t \] | \[ \frac{\pi}{j} \left[ \delta(\omega - \omega_0) - \delta(\omega + \omega_0) \right] \] | \[ a_1 = -a_{-1} = \frac{1}{2j} \]
|  |  | \[ a_k = 0, \text{ otherwise} \] |
| \[ x(t) = 1 \] | \[ 2\pi \delta(\omega) \] | \[ a_0 = 1, a_k = 0, k \neq 0 \] (This is the Fourier series representation for any choice of \( T > 0 \))

**Periodic square wave**

\[ x(t) = \begin{cases} 
1, & |t| < T_1 \\
0, & T_1 < |t| \leq \frac{T}{2}
\end{cases} \]

And \( x(t + T) = x(t) \)

\[ \sum_{n=-\infty}^{+\infty} \delta(t - nT) \]

\[ \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right) \]

\[ a_k = \frac{1}{T} \text{ for all } k \]

\[ x(t) = \begin{cases} 
1, & |t| < T_1 \\
0, & |t| > T_1
\end{cases} \]

\[ \frac{2\sin \omega T_1}{\omega} \]

\[ \frac{\omega_0 T_1}{\pi} \sin c\left(\frac{k \omega_0 T_1}{\pi}\right) = \frac{\sin k \omega_0 T_1}{k\pi} \]

\[ \frac{\sin \omega T}{\pi t} \]

\[ X(j\omega) = \begin{cases} 
1, & |\omega| < W \\
0, & |\omega| > W
\end{cases} \]

\[ \delta(t) \]

\[ \frac{1}{j\omega} + \pi \delta(\omega) \]

\[ u(t) \]
<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta(t-t_0)$</td>
<td>$e^{-j\omega t_0}$</td>
</tr>
<tr>
<td>$e^{-at}u(t), \Re{a} &gt; 0$</td>
<td>$\frac{1}{a + j\omega}$</td>
</tr>
<tr>
<td>$te^{-at}u(t), \Re{a} &gt; 0$</td>
<td>$\frac{1}{(a + j\omega)^2}$</td>
</tr>
<tr>
<td>$\frac{(-1)^n}{(n-1)!}e^{-at}u(t), \Re{a} &gt; 0$</td>
<td>$\frac{1}{(a + j\omega)^n}$</td>
</tr>
</tbody>
</table>

### Properties of the Discrete-Time Fourier Transform

\[
x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n} d\omega
\]

\[
X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}
\]

<table>
<thead>
<tr>
<th>Property</th>
<th>Aperiodic Signal</th>
<th>Fourier Transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity</td>
<td>$ax[n] + by[n]$</td>
<td>$aX(e^{j\omega}) + bY(e^{j\omega})$</td>
</tr>
<tr>
<td>Time-shifting</td>
<td>$x[n-n_0]$</td>
<td>$e^{-j\omega n_0}X(e^{j\omega})$</td>
</tr>
<tr>
<td>Frequency shifting</td>
<td>$e^{j\omega_0 n}x(n)$</td>
<td>$X(e^{j(\omega_0-\omega_0)})$</td>
</tr>
<tr>
<td>Conjugation</td>
<td>$x^*[n]$</td>
<td>$X^*(e^{-j\omega})$</td>
</tr>
<tr>
<td>Time-Reversal</td>
<td>$x[-n]$</td>
<td>$X(e^{-j\omega})$</td>
</tr>
<tr>
<td>Time Expansions</td>
<td>$x_{(k)}[n] = \begin{cases} x[n/k], &amp; \text{if } n \text{ multiple of } k \ 0, &amp; \text{if } n \neq \text{multiple of } k \end{cases}$</td>
<td>$X(e^{jk\omega})$</td>
</tr>
<tr>
<td>Convolution</td>
<td>$x[n]*y[n]$</td>
<td>$X(e^{j\omega})Y(e^{j\omega})$</td>
</tr>
<tr>
<td>Multiplication</td>
<td>$x[n]y[n]$</td>
<td>$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{i\theta})Y(e^{i(\omega-\theta)}) d\theta$</td>
</tr>
</tbody>
</table>
### Differentiation in Time

<table>
<thead>
<tr>
<th>$x[n] - x[n-1]$</th>
<th>$(1 - e^{-j\omega})X(e^{j\omega})$</th>
</tr>
</thead>
</table>

### Accumulation

| $\sum_{k=-\infty}^{n} x[k]$ | $\frac{1}{1-e^{-j\omega}}X(e^{j\omega}) + \pi X(e^{j\omega}) \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$ |

### Differentiation in Frequency

| $n x[n]$ | $\frac{j}{\omega} dX(e^{j\omega})$ |

### Conjugate Symmetry for Real Signals

<table>
<thead>
<tr>
<th>$x[n]$ real</th>
<th>$X(e^{j\omega}) = X^*(e^{-j\omega})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Re{X(e^{j\omega})} = \Re{X(e^{-j\omega})}$</td>
</tr>
<tr>
<td></td>
<td>$\Im{X(e^{j\omega})} = -\Im{X(e^{-j\omega})}$</td>
</tr>
<tr>
<td></td>
<td>$</td>
</tr>
<tr>
<td></td>
<td>$\angle X(e^{j\omega}) = -\angle X(e^{-j\omega})$</td>
</tr>
</tbody>
</table>

### Symmetry for Real and Even Signals

| $x[n]$ Real and even | $X(e^{j\omega})$ Real and Even |

### Symmetry for Real and Odd Signals

| $x[n]$ Real and odd | $X(e^{j\omega})$ Purely imaginary and odd |

### Even-Odd Decomposition for Real Signals

<table>
<thead>
<tr>
<th>$x_e[n] = \text{Ev}{x[n]}$</th>
<th>$[x[n]\text{real}]$</th>
<th>$\Re{X(e^{j\omega})}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0[n] = \text{Od}{x[n]}$</td>
<td>$[x[n]\text{real}]$</td>
<td>$\Im{X(e^{j\omega})}$</td>
</tr>
</tbody>
</table>

### Parseval’s Relation for Aperiodic Signals

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

### Properties of the Laplace Transform:

<table>
<thead>
<tr>
<th>Property</th>
<th>Signal</th>
<th>Transform</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity</td>
<td>$a x_1(t) + b x_2(t)$</td>
<td>$a X_1(s) + b X_2(s)$</td>
<td>At least $R_1 \cap R_2$</td>
</tr>
<tr>
<td>Time shifting</td>
<td>(x(t-t_0))</td>
<td>(e^{-st_0}X(s))</td>
<td>(R)</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Shifting in the s-Domain</td>
<td>(e^{st}x(t))</td>
<td>(X(s-s_0))</td>
<td>Shifted version of (R) [i.e. (s) is in the ROC if ((s-s_0)) is in (R)]</td>
</tr>
<tr>
<td>Time scaling</td>
<td>(x(at))</td>
<td>(\frac{1}{</td>
<td>a</td>
</tr>
<tr>
<td>Conjugation</td>
<td>(x^\ast(t))</td>
<td>(X^\ast\left(s^\ast\right))</td>
<td>(R)</td>
</tr>
<tr>
<td>Convolution</td>
<td>(x_1(t)\ast x_2(t))</td>
<td>(X_1(s)X_2(s))</td>
<td>At least (R_1 \cap R_2)</td>
</tr>
<tr>
<td>Differentiation in the Time Domain</td>
<td>(\frac{d}{dt}x(t))</td>
<td>(sX(s))</td>
<td>At least (R)</td>
</tr>
<tr>
<td>Differentiation in the s-Domain</td>
<td>(-tx(t))</td>
<td>(\frac{d}{ds}X(s))</td>
<td>(R)</td>
</tr>
<tr>
<td>Integration in the Time Domain</td>
<td>(\int_{-\infty}^{t}x(\tau)d\tau)</td>
<td>(\frac{1}{s}X(s))</td>
<td>At least (R \cap {\Re{s} &gt; 0})</td>
</tr>
</tbody>
</table>

**Initial and Final Value Theorems**

- If \(x(t)=0\) for \(t<0\) and \(x(t)\) contains no impulses or higher-order singularities at \(t=0\), then
  
  \[x(0^+) = \Lt_{s \to 0} s X(s)\]

- If \(x(t)=0\) for \(t<0\) and \(x(t)\) has a finite limit as \(t \to \infty\), then
  
  \[\Lt_{t \to \infty} x(t) = \Lt_{s \to 0} s X(s)\]

**Laplace Transforms of Elementary Functions**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Signal</th>
<th>Transform</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\delta(t))</td>
<td>1</td>
<td>All (s)</td>
</tr>
<tr>
<td>2</td>
<td>(u(t))</td>
<td>(\frac{1}{s})</td>
<td>(\Re{s} &gt; 0)</td>
</tr>
<tr>
<td>3</td>
<td>(-u(-t))</td>
<td>(\frac{1}{s})</td>
<td>(\Re{s} &lt; 0)</td>
</tr>
<tr>
<td></td>
<td>Sequence</td>
<td>Transform</td>
<td>ROC</td>
</tr>
<tr>
<td>---</td>
<td>----------</td>
<td>-----------</td>
<td>-----</td>
</tr>
<tr>
<td>4</td>
<td>$t^{n-1}u(t)$</td>
<td>$\frac{1}{s^n}$</td>
<td>$\Re{s} &gt; 0$</td>
</tr>
<tr>
<td>5</td>
<td>$-t^{n-1}u(-t)$</td>
<td>$\frac{1}{s^n}$</td>
<td>$\Re{s} &lt; 0$</td>
</tr>
<tr>
<td>6</td>
<td>$e^{-at}u(t)$</td>
<td>$\frac{1}{s + \alpha}$</td>
<td>$\Re{s} &gt; -\alpha$</td>
</tr>
<tr>
<td>7</td>
<td>$-e^{-at}u(-t)$</td>
<td>$\frac{1}{s + \alpha}$</td>
<td>$\Re{s} &lt; -\alpha$</td>
</tr>
<tr>
<td>8</td>
<td>$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$</td>
<td>$\frac{1}{(s + \alpha)^n}$</td>
<td>$\Re{s} &gt; -\alpha$</td>
</tr>
<tr>
<td>9</td>
<td>$-\frac{t^{n-1}}{(n-1)!}e^{-at}u(-t)$</td>
<td>$\frac{1}{(s + \alpha)^n}$</td>
<td>$\Re{s} &lt; -\alpha$</td>
</tr>
<tr>
<td>10</td>
<td>$\delta(t-T)$</td>
<td>$e^{-sT}$</td>
<td>Alls</td>
</tr>
<tr>
<td>11</td>
<td>$[\cos \omega_0 t]u(t)$</td>
<td>$\frac{s}{s^2 + \omega_0^2}$</td>
<td>$\Re{s} &gt; 0$</td>
</tr>
<tr>
<td>12</td>
<td>$[\sin \omega_0 t]u(t)$</td>
<td>$\frac{\omega_0}{s^2 + \omega_0^2}$</td>
<td>$\Re{s} &gt; 0$</td>
</tr>
<tr>
<td>13</td>
<td>$[e^{-at} \cos \omega_0 t]u(t)$</td>
<td>$\frac{s + \alpha}{(s + \alpha)^2 + \omega_0^2}$</td>
<td>$\Re{s} &gt; -\alpha$</td>
</tr>
<tr>
<td>14</td>
<td>$[e^{-at} \sin \omega_0 t]u(t)$</td>
<td>$\frac{\omega_0}{(s + \alpha)^2 + \omega_0^2}$</td>
<td>$\Re{s} &gt; -\alpha$</td>
</tr>
<tr>
<td>15</td>
<td>$u_n(t) = \frac{d^n\delta(t)}{dt^n}$</td>
<td>$s^n$</td>
<td>Alls</td>
</tr>
<tr>
<td>16</td>
<td>$u_{-n}(t) = u(t) * \ldots * u(t)$</td>
<td>$\frac{1}{s^n}$</td>
<td>$\Re{s} &gt; 0$</td>
</tr>
</tbody>
</table>

### Properties of the z-Transform

<table>
<thead>
<tr>
<th>Property</th>
<th>Sequence</th>
<th>Transform</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x[n]$</td>
<td>$X(z)$</td>
<td>$R$</td>
</tr>
<tr>
<td></td>
<td>$x_1[n]$</td>
<td>$X_1(z)$</td>
<td>$R_1$</td>
</tr>
<tr>
<td></td>
<td>$x_2[n]$</td>
<td>$X_2(z)$</td>
<td>$R_2$</td>
</tr>
<tr>
<td>Property</td>
<td>Mathematical Expression</td>
<td>z-Domain Effect</td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td>--------------------------</td>
<td>--------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td><strong>Linearity</strong></td>
<td>$a x_1[n] + bx_2[n]$</td>
<td>$aX_1(z) + bX_2(z)$ At least the intersection of $R_1$ and $R_2$</td>
<td></td>
</tr>
<tr>
<td><strong>Time shifting</strong></td>
<td>$x[n - n_0]$</td>
<td>$z^{-n_0}X(z)$ R except for the possible addition or deletion of the origin</td>
<td></td>
</tr>
<tr>
<td><strong>Scaling in the z-Domain</strong></td>
<td>$a^n x[n]$</td>
<td>$X(a^{-1}z)$ Scaled version of R (i.e., $</td>
<td>a</td>
</tr>
<tr>
<td><strong>Time reversal</strong></td>
<td>$x[-n]$</td>
<td>$X(z^{-1})$ Inverted R (i.e., $R^{-1} = { z^{-1} \mid z \in R }$)</td>
<td></td>
</tr>
<tr>
<td><strong>Time expansion</strong></td>
<td>$x(k)[n] = \begin{cases} x[r], &amp; n = rk \ 0, &amp; n \neq rk \end{cases}$</td>
<td>$X(z^k)$ R$^{1/k}$ (i.e., the set of points $z^{1/k}$ where $z \in R$)</td>
<td></td>
</tr>
<tr>
<td><strong>Conjugation</strong></td>
<td>$x^*[n]$</td>
<td>$X^<em>(z^</em>)$ R</td>
<td></td>
</tr>
<tr>
<td><strong>Convolution</strong></td>
<td>$x_1[n] * x_2[n]$</td>
<td>$X_1(z)X_2(z)$ At least the intersection of $R_1$ and $R_2$</td>
<td></td>
</tr>
<tr>
<td><strong>First difference</strong></td>
<td>$x[n] - x[n-1]$</td>
<td>$(1 - z^{-1})X(z)$ At least the intersection of $R$ and $</td>
<td>z</td>
</tr>
<tr>
<td><strong>Accumulation</strong></td>
<td>$\sum_{k=-\infty}^{n} x[k]$</td>
<td>$\frac{1}{1 - z^{-1}}X(z)$ At least the intersection of $R$ and $</td>
<td>z</td>
</tr>
<tr>
<td><strong>Differentiation in the z-Domain</strong></td>
<td>$nx[n]$</td>
<td>$-z\frac{dX(z)}{dz}$ R</td>
<td></td>
</tr>
</tbody>
</table>
Initial Value Theorem

If \( x[n] = 0 \) for \( n < 0 \), then

\[
x[0] = \lim_{z \to \infty} X(z)
\]

- Some Common z-Transform Pairs

<table>
<thead>
<tr>
<th>S.No</th>
<th>Signal</th>
<th>Transform</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \delta[n] )</td>
<td>1</td>
<td>All ( z )</td>
</tr>
<tr>
<td>2</td>
<td>( u[n] )</td>
<td>( \frac{1}{1-z^{-1}} )</td>
<td>(</td>
</tr>
<tr>
<td>3</td>
<td>( u[-n-1] )</td>
<td>( \frac{1}{1-z^{-1}} )</td>
<td>(</td>
</tr>
<tr>
<td>4</td>
<td>( \delta[n-m] )</td>
<td>( z^{-m} )</td>
<td>All ( z ) except 0 (if ( m &gt; 0 )) or ( \infty ) (if ( m &lt; 0 ))</td>
</tr>
<tr>
<td>5</td>
<td>( \alpha^n u[n] )</td>
<td>( \frac{1}{1-\alpha z^{-1}} )</td>
<td>(</td>
</tr>
<tr>
<td>6</td>
<td>( -\alpha^n u[-n-1] )</td>
<td>( \frac{1}{1-\alpha z^{-1}} )</td>
<td>(</td>
</tr>
<tr>
<td>7</td>
<td>( n\alpha^n u[n] )</td>
<td>( \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2} )</td>
<td>(</td>
</tr>
<tr>
<td>8</td>
<td>( -n \alpha^n u[-n-1] )</td>
<td>( \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2} )</td>
<td>(</td>
</tr>
<tr>
<td>9</td>
<td>( \cos\omega_0 n u[n] )</td>
<td>( \frac{1-[\cos\omega_0]z^{-1}}{1-2\cos\omega_0 z^{-1}+z^{-2}} )</td>
<td>(</td>
</tr>
<tr>
<td>10</td>
<td>( \sin\omega_0 n u[n] )</td>
<td>( \frac{[\sin\omega_0]z^{-1}}{1-2\cos\omega_0 z^{-1}+z^{-2}} )</td>
<td>(</td>
</tr>
<tr>
<td>11</td>
<td>( r^n \cos\omega_0 n u[n] )</td>
<td>( \frac{1-[r\cos\omega_0]z^{-1}}{1-2r\cos\omega_0 z^{-1}+r^2 z^{-2}} )</td>
<td>(</td>
</tr>
<tr>
<td>12</td>
<td>( r^n \sin\omega_0 n u[n] )</td>
<td>( \frac{[r\sin\omega_0]z^{-1}}{1-2r\cos\omega_0 z^{-1}+r^2 z^{-2}} )</td>
<td>(</td>
</tr>
</tbody>
</table>
Discrete Fourier Transform (DFT)

\[ X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi kn}{N}} \Rightarrow DFT \]

Where \( k = 0 \) to \( N - 1 \)

Inverse DFT is given by

\[ x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi kn}{N}} \Rightarrow IDFT \]

Where \( n = 0 \) to \( N - 1 \)

Alternative form of DFT:

\[ X(k) = \sum_{n=0}^{N-1} x(n)W^{kn} \quad \Rightarrow \quad W = e^{-j\frac{2\pi}{N}} \]

\[ x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)W^{-kn} \]

Periodicity of DFT: \( X(k + N) = X(k) \)

Matrix Formulation of DFT:

DFT in matrix form: \( X = Wx \)

Inverse DFT form: \( x = \frac{1}{N} W^H X \)

Two-point:

The two-point DFT is a simple case, in which the first entry is the DC (sum) and the second entry is the AC (difference).

\[ W = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \]

Four-point:

The four-point DFT matrix is as follows:

\[ W = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \]

Properties of DFT:

\[ g(n) \overset{DFT}{\longleftrightarrow} G(k) \]

\[ h(n) \overset{DFT}{\longleftrightarrow} H(k) \]

1. Linearity:

\( \alpha g(n) + \beta h(n) \overset{DFT}{\longleftrightarrow} \alpha G(k) + \beta H(k) \)

2. Circular shift:

\( g(n - n_0) \overset{DFT}{\longleftrightarrow} W_{N}^{k_n}G(k) \)

\( W_{N}^{-k_n}g(n) \overset{DFT}{\longleftrightarrow} G(k - k_0) \)

3. Circular convolution:

\( \sum_{m=0}^{N-1} g(m)h(n - m) \overset{DFT}{\longleftrightarrow} G(k)H(k) \)

4. Modulation:

\( g(n)h(n) \overset{DFT}{\longleftrightarrow} \left( \frac{1}{N} \right) \sum_{m=0}^{N-1} G(m)H(k - m) \)

5. Duality:

\( x(n) \overset{DFT}{\longleftrightarrow} X(k) \quad \text{then} \quad X(n) \overset{DFT}{\longleftrightarrow} N x[-k] \)

6. Parseval's relation:

\( \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2 \)
Communications

Formulas
Amplitude Modulation

- Carrier wave \( c(t) = A_c \cos (2\pi f_c t) \)
- Baseband signal \( m(t) \)
- Modulated wave
  \[ s(t) = A_c [1 + K_a m(t)] \cos (2\pi f_c t) \]
  \( K_a \) is Amplitude sensitivity
- Band Width = \( 2\omega_m \) rad/sec
  \( = 2f_m \) Hz
- \( f_m \) is message signal frequency
- **Single tone Modulation:**
  \( m(t) = A_m \cos 2\pi f_m t \)
  \( c(t) = A_c \cos 2\pi f_c t \)
  \( s(t) = A_c [1 + K_a A_m \cos 2\pi f_m t] \cos (2\pi f_c t) \)
  \( s(t) = A_c [1 + \mu \cos 2\pi f_m t] \cos (2\pi f_c t) \)
  \( \mu = K_a A_m \) → modulation index
  modulation index \( \mu = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \)
  \( P_t = P_c \left( 1 + \frac{\mu^2}{2} \right) \)
  \( I_t = \sqrt{I_c \left( 1 + \frac{\mu^2}{2} \right)} \)
  \( A_m = \frac{A_{max} - A_{min}}{2} \)

\[ A_c = \frac{A_{max} + A_{min}}{2} \]

**Efficiency** = \( \eta = \frac{\text{Useful Power}}{\text{Total Power}} \times 100 \)
\[ \eta = \frac{\mu^2}{2 + \mu^2} \times 100 \]

- **Multi tone Modulation:**
  \( P_t = P_c \left( 1 + \frac{\mu_t^2}{2} \right) \)
  \( I_t = \sqrt{I_c \left( 1 + \frac{\mu_t^2}{2} \right)} \)
  \( \mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \cdots + \mu_n^2} \)
  BW = \( 2 \times \max(\omega_{m1}, \omega_{m2}, \ldots, \omega_{mn}) \) rad/sec
  BW = \( 2 \times \max(f_{m1}, f_{m2}, \ldots, f_{mn}) \) Hz

➤ DSB-SC

Modulated Wave \( s(t) = A_c m(t) \cos (2\pi f_c t) \)
\[ S(f) = \frac{1}{2} A_c [M(f + f_c) + M(f - f_c)] \]
B.W = Same As AM
\[ P_t = P_c \mu^2 = \frac{A_m^2 A_c^2}{4} \]
Efficiency = 100%

➤ SSB

\[ s(t) = \frac{1}{2} A_c m(t) \cos (2\pi f_c t) \]
\[ \pm \frac{1}{2} A_c \bar{m}(t) \sin (2\pi f_c t) \]
+ for upper sideband, - for lower sideband
Uses product modulator then band pass filter
B.W = \( f_m \) Hz
\[ P_l = \frac{A_m^2 A_c^2}{8} \]

- **VSB**
  - Suppresses one sideband
  - Used in TV Signals Transmission.
  - Instead of using band pass filter, uses filter \( H(f) \)
  - Odd symmetry around \( f_c \)
  - Linear phase Response

- **Angle Modulation**
  - Modulated wave
    \[ s(t) = A_c \cos \left[ \theta_i(t) \right] \]
    \( \theta_i(t) \) Angle of a modulated carrier
  - \( A_c \) Carrier Amplitude
  - Instantaneous Frequency
  - \( \lim_{\Delta t \to 0} f_{\Delta t}(t) = \frac{1}{2\pi} \frac{d \theta_i(t)}{dt} \)

- **Phase Modulation**
  \[ \theta_i(t) = 2\pi f_c t + k_p m(t) \]
  \[ s(t) = A_c \cos \left[ 2\pi f_c t + k_p m(t) \right] \]
  - \( 2\pi f_c t \) Angle of un modulated signal
  - \( k_p \) Phase sensitivity
  - Let \( m(t) = A_m \cos 2\pi f_m t \)
  - Peak Phase deviation=\( \beta_{PM} = K_p A_m \)
  - Modulation index=\( \beta_{PM} = K_p A_m \)

- **Frequency Modulation**
  \[ f_i(t) = f_c + k_f m(t) \]
  \[ s(t) = A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right] \]
  \( f_c \) Frequency of unmodulated carrier
  \( k_f \) Frequency Sensitivity

  Let, \( m(t) = A_m \cos (2\pi f_m t) : \]

\[ f_i(t) = f_c + k_f m(t) \]
\[ = f_c + k_f A_m \cos (2\pi f_m t) \]
\[ = f_c + \Delta f \cos (2\pi f_m t) \]
\[ \Delta f = k_f A_m \rightarrow \text{frequency Deviation;} \]
Instantaneous phase
\[ \theta_i(t) = 2\pi \int_0^t f_i(\tau) d\tau \]
\[ = 2\pi f_c t + \frac{\Delta f}{f_m} \sin (2\pi f_m t) \]
\[ = 2\pi f_c t + \beta \sin (2\pi f_m t) \]

- **\( \beta \)** Modulation Index;

- **FM signal**
  \[ s(t) = A_c \cos \left[ 2\pi f_c t + \beta \sin (2\pi f_m t) \right] \]
  \[ \beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m} \]

- **\( \beta < 1\text{radian} \)** Narrowband FM
  \[ s(t) \approx A_c \cos (2\pi f_c t) \]
  \[- A_c \beta \sin (2\pi f_m t) \sin (2\pi f_c t) \]
  Band Width=\( 2f_m \) Hz
  Total power: \( p_t = \frac{A_c^2}{2} (1 + \beta^2) \)

- **\( \beta > 1\text{radian} \)** Wideband FM
  \[ s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos [2\pi (f_c + n f_m) t] \]
  Bandwidth (from carson’s rule)
  \[ B_T \approx 2\Delta f + 2f_m = 2f_m (1 + \beta) \]
  Total Power: \( p_t = \frac{A_c^2}{2} \)

- **Figure of merit (FOM):**
  \[ FOM = \frac{SNR_{o/p}}{SNR_{i/p}} = \frac{1}{\text{Noise Figure}} \]

  - For SSB-SC and DSB-SC : FOM=1
Digital Communications

- **Sampling:** (continuous to discrete time)
  \[ f_s \geq 2f_m \]
  - Nyquist rate: \( f_s = \frac{1}{T_s} = 2f_m \)
  - Under sampling: \( f_s < 2f_m \)
  - Oversampling: \( f_s > 2f_m \)

- **Pulse Code Modulation:**
  
  Step Size: \( \Delta = \frac{V^+ - V^-}{L} = \frac{V_{pp}}{L} = \frac{V_{pp}}{2^n} \)
  
  No. of Levels: \( L = 2^n \)
  
  \([Q_e] = \text{sampled value} - \text{quantize value}\)
  
  \([Q_e]_{\text{max}} = \frac{\Delta}{2}\)
  
  Bit rate: \( R_b = \frac{1}{T_b} = nf_s = \frac{n}{T_s} \)
  
  \((B.W)_{\text{max}} = R_b; (B.W)_{\text{min}} = \frac{R_b}{2}\)
  
  Signal to Quantization Noise Ratio (SQNR):
  
  \( SQNR = \frac{3}{2}2^{2n} \)
  
  \( SQNR \text{ in dB} = (6n + 1.76)_{\text{dB}} \)

- **DPCM:**
  
  - Bit rate of PCM and DPCM are same.
  
  - Quantization error is less in DPCM as compared to PCM. Because we encode difference two successive samples.

- **Delta Modulation:**
  
  - Error Signale[n] = \( m[n] - m_q[n-1] \)

- **Quantized:** \( e_q[n] = \Delta sgn(e[n]) \)

- **Output of DM:** \( m_q[n] = m_q[n-1] + e_q[n] \)

- **Slope Overload Distortion occurs when**
  
  \[ \frac{\Delta}{T_s} < \max \left| \frac{dm(t)}{dt} \right| \]

- **Granular Noise occurs when**
  
  \[ \frac{\Delta}{T_s} > \max \left| \frac{dm(t)}{dt} \right| \]

- **Optimal step size:** \( \Delta_{\text{optimum}} \)
  
  \[ \Delta_{\text{optimum}} = T_s \times \max \left| \frac{dm(t)}{dt} \right| \]

- **Time Division Multiplexing (TDM):**
  
  \( R_b = \frac{N_n}{T_s} = NnT_s \)

- **Information Theory:**

  Consider a DMS, denoted by \( X \), with alphabets \( \{x_1, x_2, \ldots, x_m\} \). And probability of occurrence of symbol \( x_i \) is \( P(x_i) \).

  Information content:
  
  \[ I(x_i) = \log_2 P(x_i) \]

  Properties of \( I(x_i) \):
  
  1. \( I(x_i) = 0 \) for \( P(x_i) = 1 \)
  2. \( I(x_i) \geq 0 \)
  3. \( I(x_i) > I(x_j) \) if \( P(x_i) < P(x_j) \)
  4. \( I(x_i x_j) = I(x_i) + I(x_j) \) if \( x_i \) and \( x_j \) are independent.

  **Average Information or Entropy:**
  
  \[ H(X) = E[I(x_i)] \]
  
  \[ = \sum_{i=1}^{m} P(x_i) \log_2 \frac{1}{P(x_i)} \text{ bits/symbol} \]
Information Rate:
If the rate at which source \( X \) emits symbols is \( r \) (symbols/s), the information rate \( R \) of the source is given by,
\[
R = rH(X) \text{ bits/sec.}
\]

Shannon’s Capacity Theorem:
Maximum rate at which information can be transmitted across the channel without error.
\[
C = B \log_2(1 + SNR)
\]
Where \( C \) is Information capacity, \( B \) is Channel bandwidth and SNR is Received SNR.

- **Probability and Random Process:**
  - **Cumulative Distribution function (CDF):**
    \[
    F_X(x) = P\{X \leq x\}
    \]
  - **Properties of CDF:**
    1. \( 0 \leq F_X(x) \leq 1 \)
    2. \( F_X(x_1) \leq F_X(x_2) \) if \( x_1 < x_2 \)
    3. \( F_X(\infty) = 1 \)
    4. \( F_X(-\infty) = 0 \)
    5. \( P(a < X \leq b) = F_X(b) - F_X(a) \)
    6. \( P(X > a) = 1 - F_X(a) \)
  - **Probability Density Function (PDF):**
    \[
    f_X(x) = \frac{dF_X(x)}{dx}
    \]
    \[
    F(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(\xi) d\xi
    \]
  - **Properties of \( f_X(x) \):**
    1. \( f_X(x) \geq 0 \)
    2. \( \int_{-\infty}^{\infty} f_X(x) dx = 1 \)
    3. \( P(a < X \leq b) = \int_{a}^{b} f_X(x) dx \)

### Mean:
\[
\mu = E\{X\} = \int_{-\infty}^{\infty} xf_X(x) dx
\]

### Variance:
\[
\sigma^2 = E\{(X - \mu)^2\} = E\{X^2\} - \mu^2
\]
For a random variable, the outcome of a random experiment is mapped into a real number.

#### Uniform Random Variable:
\[
f_X(x) = \begin{cases} 
\frac{1}{b-a}; & a < x \leq b \\
0; & \text{else}
\end{cases}
\]
Mean: \( \mu = \frac{a+b}{2} \)
Variance: \( \sigma^2 = \frac{(b-a)^2}{12} \)

#### Gaussian Random Variable:
\[
f_X(x) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]
\( X \sim N(\mu, \sigma^2) \)
Where \( \mu = \text{mean} \) and \( \sigma^2 = \text{variance} \)
- For a random variable, the outcome of a random experiment is mapped into a number.
- For a random process, outcome of a random experiment is mapped into a wave form that is a function of time.

#### Statistical Averages [Ensemble Averages]:
- **Mean:**
  \[
  \mu_X(t) = E[X(t)] = \int_{-\infty}^{\infty} x f_X(x; t) dx
  \]
- **Auto Correlation:**
  \[
  R_{XX}(t_1, t_2) = E[X(t_1)X(t_2)]
  \]
  \[
  R_{XX}(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1 x_2 f_X(x_1, x_2; t_1 t_2) dx_1 dx_2
  \]
Stationarity:

1. **Strict -Sense Stationary:**

   A random process $X(t)$ is called SSS if its statistics are invariant to a shift of origin. i.e.
   
   $$f_X(x_1,\ldots,x_n;t_1,\ldots,t_n) = f_X(x_1,\ldots,x_n;t_1 + c,\ldots,t_n + c)$$ for any $C$

2. **Wide -Sense Stationary:**

   A random process $X(t)$ is called WSS if its mean is constant and auto correlation depends only on the time difference $\tau$.

   $$E[X(t)] = \mu_X$$

   $$E[X(t)X(t + \tau)] = R_{XX}(\tau)$$

**Time Averages:**

- Time -averaged mean of a sample function $x(t)$ of a random process $X(t)$ is defined as,

  $$\bar{x} = \langle x(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)dt$$

- Time -averaged autocorrelation of a sample function $x(t)$ is defined as,

  $$\bar{R}_{XX}(\tau) = \langle x(t)x(t + \tau) \rangle$$

  $$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t + \tau)dt$$

**Ergodicity:** A random process is ergodic, if time averages are equal to ensemble averages.

**Auto Correlation Function and Power Spectral Density:**

1. Auto Correlation Function $R_{XX}(\tau)$:

   $$R_{XX}(\tau) = E[X(t)X(t + \tau)]$$

**Properties of ACF:**

1. $R_{XX}(-\tau) = R_{XX}(\tau)$
2. $|R_{XX}(\tau)| \leq R_{XX}(0)$
3. $R_{XX}(0) = E[X^2(t)]$

2. **Power Spectral Density:**

   Fourier Transform of Auto Correlation Function $R_{XX}(\tau)$ is called Power Spectral Density $S_{XX}(\omega)$.

**Wiener-Khinchin Relations:**

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)e^{-j\omega \tau}d\tau$$

$$R_{XX}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX}(\omega)e^{j\omega \tau}d\omega$$

**Properties of $S_{XX}(\omega)$:**

1. $S_{XX}(\omega)$ is real and $S_{XX}(\omega) \geq 0$
2. $S_{XX}(-\omega) = S_{XX}(\omega)$
3. $\frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX}(\omega)d\omega = R_{XX}(0) = E[X^2(t)]$

**Transmission of Random Process Through LTI system:**

- Let $h(t)$ be the impulse response of LTI system. Random Process $X(t)$ is input to $h(t)$, output random process is $Y(t)$.

  $$Y(t) = X(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)X(t - \tau)d\tau$$

**Mean of output:**

$$\mu_Y(t) = E[Y(t)] = \mu_X(t)H(0)$$

**Power Spectral Density of Output:**

$$S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$$

**Auto Correlation of output:**

$$R_{YY}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{YY}(\omega)e^{j\omega \tau}d\omega$$
Digital Carrier Transmission:

Phase Shift Keying (PSK)

\[ s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \rightarrow \text{symbol '1'} \]
\[ s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) \rightarrow \text{symbol '0'} \]

Energy per bit

\[ E_b = \text{Energy per bit} \]

Phase angle

\[ \phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \quad 0 \leq t \leq T_b \]

Band Width

\[ B.W = 2R_b = \frac{2}{T_b} \]

Probability of error

\[ P_b = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{2N_0}} \right) \]

Where,

\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{t^2}{2}} dt = \frac{1}{2} \text{erfc} \left( \frac{x}{\sqrt{2}} \right), x \geq 0 \]

Amplitude Shift Keying (ASK): (On Off Keying)

\[ s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \rightarrow \text{symbol '1'} \]
\[ s_2(t) = 0 \rightarrow \text{symbol '0'} \]

\[ \text{Band Width} = 2R_b = \frac{2}{T_b} \]

\[ \text{Probability of error} P_e = Q \left( \sqrt{\frac{E_b}{2N_0}} \right) \]

Frequency Shift Keying (FSK):

\[ s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t) \rightarrow \text{symbol '1'} \]
\[ s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_0 t) \rightarrow \text{symbol '0'} \]

\[ f_1 = f_c + \Delta f; f_0 = f_c - \Delta f \]

\[ \text{Band Width} = (f_1 - f_0) + \frac{2}{T_b} = 2\Delta f + 2R_b \]

\[ \text{Probability of error} P_e = Q \left( \sqrt{\frac{E_b}{N_0}} \right) \]
Electro Magnetics
Formulas
Unit 9

Electro Magnetics

Electrostatics

\[ B = \mu H \; ; \; D = \varepsilon E \]

\[ Q = \text{point charge} = \rho_l \, dl = \rho_v \, ds = \rho_d \, dv \]

\[ l \, d \bar{l} = \text{point current element} = \bar{K} \, ds \]

\[ E = \frac{Q}{4\pi \varepsilon r^2} \, a_r \; ; \; \rho = \frac{\rho_L}{2\pi \varepsilon} \, a_r \; ; \; \frac{\rho_s}{2\varepsilon} \, a_N \]

\[ H = \frac{I \, d \bar{l}}{4\pi r^2} \times a_r = \frac{I}{2\pi \varepsilon} \, a_\phi = \frac{K}{2} \, a_N \]

\[ V = \frac{W}{Q} = \text{Joules} / \text{Coulombs} \]

\[ V = -\int E \cdot d \bar{l} = \text{potential function} \]

\[ V_{AB} = -\int_B^A \bar{E} \, d \bar{l} = \text{potential difference} \]

\[ E = -\nabla V = \text{potential gradient} \]

\[ \bar{A} = \text{vector magnetic potential} \]

\[ = \text{weber/meter} \]

\[ \bar{B} = \nabla \times \bar{A} \]

\[ V = \frac{Q}{4\pi \varepsilon r} ; \; \bar{A} = \mu I \, d \bar{l} \times \frac{a_r}{4\pi r} \]

Maxwell’s Equations (general differential)

\[ \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \]

\[ \nabla \cdot \bar{D} = \bar{\rho} \]

\[ \nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \]

\[ \nabla \cdot \bar{B} = 0 \]

Maxwell’s Equations (time harmonic)

\[ \nabla \times \bar{E} = -j\omega \bar{B} \]

\[ \nabla \cdot \bar{D} = \bar{\rho} \]

\[ \nabla \times \bar{H} = \bar{J} + j\omega \bar{D} = \sigma \bar{E} + j\omega \varepsilon \bar{E} \]

\[ \nabla \cdot \bar{B} = 0 \]

Maxwell’s Equations (integral)

\[ \oint_c \bar{E} \cdot d \bar{l} = -\int_s \frac{\partial \bar{B}}{\partial t} \cdot d \bar{s} \]

\[ \oint_s \bar{D} \cdot d \bar{s} = \int_v \bar{\rho} d \bar{v} \]

\[ \oint_c \bar{H} \cdot d \bar{l} = \int_s \bar{J} \cdot d \bar{s} + \int_s \frac{\partial \bar{D}}{\partial t} \cdot d \bar{s} \oint_s \bar{B} \cdot d \bar{s} = 0 \]

Continuity Equation:

\[ \nabla \cdot \bar{J} = -\frac{\partial \bar{\rho}_v}{\partial t} \]

Ohm’s law in point form: \( J = \sigma E \)

\[ \bar{\rho}_v(t) = \rho_v \, e^{-\frac{\sigma t}{\varepsilon}} \]

\[ \nabla^2 V = -\frac{\bar{\rho}_v}{\varepsilon} \rightarrow \text{poisson’s equation} \]

\[ \nabla^2 V = 0 \rightarrow \text{Laplace equation} \]

Boundary Conditions:

Dielectric -to- Dielectric:

\( E_{t1} = E_{t2}, B_{n1} = B_{n2} \)

\( D_{n2} - D_{n1} = \rho_s \rightarrow D_{n1} = D_{n2} \)

\( H_{t2} - H_{t1} = \bar{K} \times a_n \rightarrow H_{t1} = H_{t2} \)

Conductors:

\( E_{\text{tang}} = 0, D_{\text{normal}} = \rho_s \)
EM Wave equations:
\[ \nabla^2 E = \mu e \frac{\partial^2 E}{\partial t^2} \]
\[ \nabla^2 H = \mu e \frac{\partial^2 H}{\partial t^2} \]

Helmholtz’s equations:
\[ \nabla^2 E = \gamma^2 E \]
\[ \nabla^2 H = \gamma^2 H \]

Propagation constant \( \gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta \)

Intrinsic Impedance:
\[ \eta = \sqrt{j\omega\mu \over \sigma + j\omega\varepsilon} \]

For free space,
\[ \gamma = j\omega\sqrt{\mu_0\varepsilon_0} ; \eta = \sqrt{\mu_0 \over \varepsilon_0} = 120\pi \]

In ideal dielectric,
\[ \gamma = j\omega\sqrt{\mu_0\varepsilon_0\varepsilon_r} ; \eta = \sqrt{\mu_0 \over \varepsilon_0\varepsilon_r} = 120\pi \sqrt{\varepsilon_r} \]

Phase velocity:
\[ v_p = \frac{\omega}{\beta} \]

In free space \( v_p = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3 \times 10^8 \text{ m/sec} \)

In dielectric \( v_p = 3 \times 10^8 / \sqrt{\varepsilon_r} \)

Loss Tangent:
\[ \delta = \frac{J_c}{J_D} = \frac{\sigma}{\omega\varepsilon} \]

If \( \frac{\sigma}{\omega\varepsilon} \gg 1 \) very good conductor,
\[ \frac{\sigma}{\omega\varepsilon} = 0 \text{ free space or ideal dielectric} \]
\[ \frac{\sigma}{\omega\varepsilon} \ll 1 \text{ lossy dielectric} \]

Plane waves in lossy materials
\[ \gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} \]
\[ \alpha = \omega\sqrt{\frac{\mu\varepsilon}{2} \left[ 1 - \left( \frac{\sigma}{\omega\varepsilon} \right)^2 - 1 \right]^{1/2}} \]
\[ \beta = \omega\sqrt{\frac{\mu\varepsilon}{2} \left[ 1 + \left( \frac{\sigma}{\omega\varepsilon} \right)^2 + 1 \right]^{1/2}} \]

Loss Less Dielectric: \( (\sigma \approx 0) \)
\[ \alpha = 0; \beta = \omega\sqrt{\mu\varepsilon} \]

Free Space: \( (\sigma = 0, \mu = \mu_0, \varepsilon = \varepsilon_0) \)
\[ \alpha = 0, \beta = \omega\sqrt{\mu_0\varepsilon_0} \]

Good Conductor:
\[ \sigma \gg \omega\varepsilon \implies \frac{\sigma}{\omega\varepsilon} \rightarrow \infty \]
\[ \alpha = \beta = \sqrt{\pi f \mu\sigma} \]
\[ \eta = \sqrt{\omega\mu \over \sigma} \angle 45^\circ \]

Skin depth: \( \delta = \frac{1}{\alpha} \)

Skin Resistance \( R_s = \frac{1}{\sigma\delta} = \sqrt{\pi f \mu \over \sigma} \)

Poynting Theorem:
\[ \int_v \mathbf{E} \cdot \mathbf{J} dv = -\frac{\partial}{\partial t} \int_v \left( \frac{1}{2} \mu |\mathbf{H}|^2 + \frac{1}{2} \varepsilon |\mathbf{E}|^2 \right) dv - \oint_s (\mathbf{E} \times \mathbf{H}) \cdot \mathbf{ds} \]
\[ P_{avg} = \frac{E_0^2}{2\eta} \]
\[ P_{avg} = \frac{1}{2} \text{Re} \{ E \times H \} \]

Wave Polarization:
Let,
\[ E(z,t) = E_{x0} \cos(\omega t - \beta z) a_x \]
\[ + E_{y0} \cos(\omega t - \beta z + \theta) a_y \]

If \( E_{x0} = 0 \) or \( E_{y0} = 0 \) or \( \theta = 0^\circ \) or \( 180^\circ \)
Linear polarization.

If $E_{x0} = E_{y0}$ and $\theta = 90^\circ$ circular polarization.

All other cases are elliptical.

**Wave Incidence:**

**Normal Incidence:**

$\Gamma = \frac{n_2 - n_1}{n_2 + n_1} = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$

$T = 1 + \Gamma$

**Oblique Incidence:**

Horizontal E polarized (s-polarized)

$\Gamma_s = \frac{n_2 \sec \theta_t - n_1 \sec \theta_l}{n_2 \sec \theta_t + n_1 \sec \theta_l}$

Vertical E polarized (p-polarized)

$\Gamma_p = \frac{n_2 \cos \theta_t - n_1 \cos \theta_l}{n_2 \cos \theta_t + n_1 \cos \theta_l}$

Critical angle: $\theta_c = \sin^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

Brewster angle: $\theta_B = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

**Transmission Lines**

- Transmission lines are structures which capable of guiding TEM waves.
  Ex: Coaxial lines, Parallel plates and Two wire lines.
- Wave Equations
  
  $\frac{\partial V(z)}{\partial z} - \gamma^2 V(z) = 0$
  
  $\frac{\partial I(z)}{\partial z} - \gamma^2 I(z) = 0$

- $\gamma$ is complex propagation constant
  
  $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

- Characteristic Impedance
  
  $Z_0 = \frac{R + j\omega L}{\sqrt{G + j\omega C}}$

- Loss Less: $R = G = 0$

- Distortionless: $\frac{R}{L} = \frac{G}{C}$

- Wave Velocity (Phase Velocity):
  
  $v_p = \frac{\omega}{\beta} = f; \quad \lambda = \frac{2\pi}{\beta}$

  $v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{LC}}$

- Input Impedance:
  
  $Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$ (lossy)

  $Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$ (loss less)

- Shorted Circuit: $Z_L = 0 \rightarrow Z_{SC} = jZ_0 \tan \beta l$

- Open Circuit: $Z_L = \infty \rightarrow Z_{OC} = -jZ_0 \cot \beta l$

- Characteristic Impedance: $Z_0 = \sqrt{Z_{SC}Z_{OC}}$

- Quarter – Wave Line:
  
  $l = \frac{\lambda}{4} \rightarrow Z_{in} = \frac{Z_0^2}{Z_L}$

- Half – Wave Line:
  
  $l = \frac{\lambda}{2} \rightarrow Z_{in} = Z_L$

- Voltage Reflection Coefficient:
  
  $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$
Special cases:
- Short: \( Z_L = 0 \rightarrow \Gamma_L = -1 \)
- Open: \( Z_L = \infty \rightarrow \Gamma_L = +1 \)
- Match: \( Z_L = Z_0 \rightarrow \Gamma_L = 0 \)

- VSWR or CSWR = \( \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{l_{\text{max}}}{l_{\text{min}}} \)
- \( SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = (1, \infty) \)

Smith Chart:
- Smith chart is used to calculate \( \Gamma \) and VSWR for normalized load impedance.
- Normalized load Impedance = \( Z_L / Z_0 \)
- It is a rectangular graph of \( \Gamma_r Vs \Gamma_i \), and it contain two family of circles.

Constant R-circles:
- center: \( \left( \frac{R}{R+1}, 0 \right) \)
- radius: \( \frac{1}{R+1} \)

Constant X-circles:
- center: \( \left( 1, \frac{1}{X} \right) \)
- radius: \( \frac{1}{X} \)

Wave Guides

TM mode: \( (H_Z = 0) \)

Propagation Constant:
\[
\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \varepsilon}
\]

Where \( \omega \sqrt{\mu \varepsilon} = k \)

Cut Off Frequency:
\[
\omega_c = 2\pi f_c = \frac{1}{\sqrt{\mu \varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}
\]

\( k^2 < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \rightarrow \text{Evansent mode} \)

\( k^2 > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \rightarrow \text{Propagation mode} \)

Evansent mode:
\[
\eta_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \beta = \frac{\mu}{\varepsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}
\]

TE Mode: \( (E_Z = 0) \)

\[
\eta_{TE} = \frac{\omega \mu}{\beta} = \frac{\mu}{\sqrt{\varepsilon}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}
\]

\( \eta_{TE} > \eta_{TM} \)

\( TE_{10} \) is dominant mode.
Engineering Mathematics

Formulas
Matrix: It is an arrangement of some Real or Complex numbers in the form of Square or Rectangle in Rows and Columns.

Types of Matrices:
1. Square Matrix: Rows and Columns are equal.
2. Rectangle Matrix: Rows and Columns are not equal.
3. Null Matrix or Zero Matrix: All elements are zero. It is denoted by O.
4. Column Matrix: It has only one column.
5. Row Matrix: It has only one row.
   Principal Diagonal: In a Square Matrix, the DIAGONAL from the first element of first row to the last element of last row.

Trace of a Square Matrix: Sum of the Principal Diagonal Elements. It is denoted by Trac(A) or tr(A).
6. Diagonal Matrix: Principal diagonal elements are non-zero and other are zero.
7. Scalar Matrix: Principal diagonal elements are equal(except 1) and other are zero.
8. Unit (Identity) Matrix: Principal diagonal elements are equal to 1 and other are zero.
9. Triangular Matrix: A Square Matrix is called “Triangular Matrix”, if it is either Lower Triangular or Upper Triangular Matrix.
   - Lower Triangular Matrix: All the elements above the principal diagonal are zero.
   - Upper Triangular Matrix: In this all the elements below the principal diagonal are zero.
10. Idempotent Matrix:
    - If A be a matrix such that A^2 = A, Then A is called Idempotent Matrix.
    - If AB = A and BA = B, then A and B are idempotent.
11. Involuntary Matrix: If A be a matrix such that A^2 = I, then A is called Involuntary Matrix.
12. Nilpotent Matrix:
    - A square matrix A is called a Nilpotent Matrix, if there exists a +ve integer n such that An = O.
    - If n is the least positive integer such that Am = O, then n is called the index of the nilpotent matrix A.
13. Orthogonal Matrix: A square matrix A is said to be an Orthogonal Matrix, if \( AA^T = A^T A = I \)
**Transpose of a Matrix:** The matrix A is obtained by interchanging its rows and columns is called the Transpose matrix of A. It is denoted by $A^T$.

**Symmetric Matrix:** $A^T = A$

**Skew-Symmetric Matrix:** $A^T = -A$
In a skew symmetric matrix, all the elements along the diagonal are zero.

**Complex Matrices:**
1. **Hermitian Matrix:** $A^θ = A$
   Where $A^θ$ is the transposed conjugates of A.
2. **Skew-Hermitian Matrix:** A square matrix A is called Skew Hermitian Matrix, if $A^θ = -A$,
   Where $A^θ = (A^T)^*$ or $(A^*)^T$ the diagonal elements of a skew-Hermitian matrix must be pure imaginary numbers or zero.
3. **Unitary Matrix:** A square matrix A is said to be Unitary Matrix, if $A^θ = A^*$.

**Determinant of a Square Matrix:** Only square matrices can have determinants because in a determinant the number of rows is equal to the number of columns.

Let $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ be an $n \times n$ square matrix.

Then

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{vmatrix}$$

Where $|A|$ is called determinant of matrix A.

**Minor of a Square Matrix:** The $(n-1)^{th}$ order determinant is called Minor of a Square Matrix. It is denoted by $|M_{ij}|$.

**Cofactor:** The Co-factor of an element $a_{ij}$ is denoted by $A_{ij}$ and is defined by $(-1)^{i+j}|M_{ij}|$.

**Adjoint Matrix:** The transpose of all the co-factors of a square matrix A is called Adjoint of A. It is denoted by “Adj A” or “adj A”.

**Inverse of Matrix:** Let A and B are two square matrices of the same order, such that $AB = BA = I$
Then B is called the inverse of A and is denoted by $A^{-1}$

$$A^{-1} = \frac{(adj A)}{|A|}$$

**Note:**

i) $\text{Adj}(AB) = (\text{Adj}A)(\text{Adj}B)$

ii) $\text{Adj}(A^m) = (\text{Adj}A)^m$

iii) $\text{Adj}(A^T) = (\text{Adj}A)^T$

iv) $\text{Adj}(KA) = k^{n-1}(\text{Adj}A)$, where $k \in \mathbb{R}$ and $A$ be a $n \times n$ matrix

v) $|AB| = |A||B|$

vi) $|kA| = k^n|A|$, where $k \in \mathbb{R}$ and $A$ be a $n \times n$ matrix

vii) $|A^{-1}| = |A|^{-1}$

viii) Inverse of a Matrix is unique.

ix) $(A^{-1})^{-1} = A$ and $(AB)^{-1} = B^{-1}A^{-1}$

x) $A^{-1}$ is exists if and only if A is Non-Singular Matrix.

xi) $(A^T)^{-1} = (A^{-1})^T$

xii) $(A^{-1})^θ = (A^θ)^{-1}$
Rank of a Matrix:

1. Determinant Method: Rank of a zero matrix is zero. Let ‘A’ be a given non-zero matrix. we say that ‘r’ is “Rank” of A, if (i) Every (r+1)th order minor is zero. (ii) There exists at least one rth order minor, which is not zero.

Rank of A is denoted by ρ(A).

Properties:
1. Rank of a Null Matrix is zero.
2. Rank of a Unit Matrix of order n is n.
3. Rank of a matrix is unique.
4. If A=B, then ρ(A) = ρ(B).
5. ρ(A)= ρ(Aᵀ)
6. ρ(AB)= Min{ ρ(A), ρ(B)}
7. If A is a singular matrix of order n, then ρ(A)<n
8. If A is a non-singular matrix of order n then ρ(A) = n.

2. Echelon form: Let A be a given non-zero matrix, we say that matrix A is in Echelon form, if A satisfies the following properties
a. Zero rows if any, are below non-zero row
b. The Number of zeros before the first non-zero element in a row is less than the No of such zeros in the next row.
ρ(A) = the no of non-zero rows

System of Linear Equations:
1. Homogeneous linear equations:
Let the system of m homogeneous equations in n unknown x₁, x₂,…,xₙ be

\[
\begin{align*}
\text{Equation (i)} & \\
\text{Equation (ii)} & \\
\end{align*}
\]

Where A, X and O are m×n, n× 1 and m× 1 matrices respectively.

Equation (i) can be written in the form of a single matrix equation

AX = O ……..(ii)

The matrix A is called the Coefficient Matrix of the system of equation (i)

Working rule:
1. Write matrix A.
2. Reduce the matrix A into Echelon form.
3. (i) If ρ(A) = n, then the given system has trivial (zero) solution.
   (ii) If ρ(A)<n, then the given system has Non-trivial (Non-zero) solution.
   i.e., There are (n – r) Linearly Independent (L.I.) solutions.
   Where r = Rank of A
   n = No. of unknowns (variables)
2. Systems of Linear Non-homogeneous Equations:

Let the system of \( m \) simultaneous equations in \( n \) unknowns \( x_1, x_2, \ldots, x_n \) be

\[
\begin{align*}
\sum_{i=1}^{n} a_{i1} x_i &= K_1 \\
\sum_{i=1}^{n} a_{i2} x_i &= K_2 \\
&\vdots \\
\sum_{i=1}^{n} a_{in} x_i &= K_m
\end{align*}
\]

Let

\[
A = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
& \cdots & \cdots & \cdots \\
a_{mn} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}
\]

Let

\[
X = \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix}, \quad B = \begin{bmatrix}
k_1 \\
k_2 \\
\vdots \\
k_m
\end{bmatrix}
\]

Where \( A, X, B \) are \( m \times n, n \times 1 \) and \( m \times 1 \) matrices respectively, then above equations can be written in the form of a single matrix equation \( AX = B \).

**Working Rule:**

1. Write Augmented Matrix \([A \ B]\).
2. Reduce \([A \ B]\) into echelon form.
3. (i) If \( \rho([A \ B]) \neq \rho(A) \), then the given system is inconsistent and has no solution.
   (ii) If \( \rho([A \ B]) = \rho(A) \) and \( \rho(A) = n \), then the given system is consistent and has a unique solution.
   (iii) If \( \rho([A \ B]) = \rho(A) \) and \( \rho(A) < n \), then the given system is consistent and have an infinite number of solutions. i.e., There are “\( n - r \)” Linearly Independent (L.I.) solutions.

**Eigen Values and Eigen Vectors**

1. Let \( A = \begin{bmatrix} a_{ij} \end{bmatrix} \) be an \( n \times n \) matrix.

   - **Characteristic Matrix of \( A \):** The matrix \( A - \lambda I \) is called the characteristic matrix of \( A \), where \( I \) is the identity matrix.

   - **Characteristic Polynomial of \( A \):** The determinant \( |A - \lambda I| \) is called the characteristic polynomial of \( A \).

   - **Characteristic Equation of \( A \):** The equation \( |A - \lambda I| = 0 \) is known as the characteristic equation of \( A \) and its roots are called the Eigen values. (or) Characteristic Roots (or) Latent Roots.

   - **Eigen value Problem:** The problem of finding the Eigen values of a matrix is known as an Eigen-value problem.

   - **Spectrum:** The set of Eigen values is called “Spectrum”.

   - **Characteristic Vector:** Let \( A \) be a square matrix of order \( n \) and \( \lambda \) be an Eigen value. If there exist a non-zero vector \( X \) such that \( AX = \lambda X \), then ‘\( X \)’ is called “Characteristic Vector”.

   - **Properties of Eigen Values and Eigen Vectors:**
     1. The Eigen values of \( A \) and \( A^T \) are same.
     2. The Eigen values of Triangular Matrix, Diagonal Matrix, Unit Matrix and Scalar
Matrix are its principal diagonal elements.
3. The Eigen values of real symmetric matrix are real.
4. The Eigen values of skew symmetric matrix are either purely imaginary or zero.
5. The Eigen values of an orthogonal matrix are 1 and -1.
6. The Eigen vectors of A and $A^{-1}$ are same.

**The Cayley-Hamilton Theorem:**
Every square matrix satisfies its own characteristic equation.

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### Calculus

#### LIMITS
1. Let function $f$ be defined in a deleted nbd of ‘a’ and $l \in \mathbb{R}$. If for each $\varepsilon > 0$ there exists $\delta > 0$ so that $0 < |x - a| < \delta \Rightarrow |f(x) - l| < \varepsilon$ then we say that $\lim_{x \to a} f(x) = l$

   - If for each $\varepsilon > 0$ there exists $\delta > 0$ so that $a - \delta < x < a \Rightarrow |f(x) - l| < \varepsilon$ then we say that $\lim_{x \to a^-} f(x) = l = f(a -)$

   - If for each $\varepsilon > 0$ there exists $\delta > 0$ so that $a < x < a + \delta \Rightarrow |f(x) - l| < \varepsilon$ then we say that $\lim_{x \to a^+} f(x) = l = f(a +)$

2. $\lim_{x \to a^-} f(x) = l \iff \lim_{x \to a^+} f(x) = l = \lim_{x \to a} f(x)$.

3. $\lim_{x \to a} f(x)$ does not exist if either
   - (a) $\lim_{x \to a^-} f(x)$ does not exist or
   - (b) $\lim_{x \to a^+} f(x)$ does not exist or
   - (c) $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ both exist and are not equal.

4. If $n \in \mathbb{N}$ then $\lim_{x \to a} x^n = a^n$.
5. If $x > 0$, $n \in \mathbb{R}$ then $\lim_{x \to a} |x|^n = a^n$.
6. $\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n$
7. $\lim_{x \to a} e^{f(x)} = e^{\lim_{x \to a} f(x)}$

   - $\lim_{x \to a} \log f(x) = \log [\lim_{x \to a} f(x)]$

   - provided $\lim_{x \to a} f(x) = 1 > 0$.
8. $\lim_{x \to a} f(x) = 1$.
9. $\lim$ Indeterminate forms: $0^0, 0^n, \infty^0, \infty \cdot \infty, 1^\infty$ etc.
10. If $a \neq 0$, $n \in \mathbb{Q}$ then $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$
11. $\lim_{x \to a} \sin x = \sin a, \lim_{x \to a} \cos x = \cos a$.
12. If $x$ is in radians and $0 < |x| < \pi/2$ then
   - a) $\lim_{x \to 0} \frac{\sin x}{x} = 1$
   - b) $\lim_{x \to 0} \frac{\tan x}{x} = 1$
   - c) $\lim_{x \to 0} \frac{\sin ax}{x} = a$
   - d) $\lim_{x \to 0} \frac{\tan ax}{x} = a$
13. If $f(x)$ is a polynomial function then $\lim_{x \to a} f(x) = f(a)$.
14. $\lim_{x \to 0} \frac{e^x - 1}{x} = 1, \lim_{x \to 0} \frac{a^x - 1}{x} = \log_a a$. 

---
16. \( \lim_{x \to 0} (1 + x)^{1/x} = e; \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e \)

17. \( \lim_{x \to 0^-} \frac{1}{x} = -\infty; \lim_{x \to 0^+} \frac{1}{x} = \infty; \lim_{x \to 0} \frac{1}{x} \) does not exist.

18. \( \lim_{x \to 0} \frac{\sin x}{x} \) does not exist.

19. \( \lim_{x \to 0^-} e^{1/x} = 0; \lim_{x \to 0^+} e^{1/x} = \infty; \lim_{x \to 0} e^{-1/x} = 0. \)

20. \( \lim_{x \to \infty} e^x = \infty; \lim_{x \to \infty} e^{-x} = 0. \)

21. \( \lim_{n \to \infty} x^n = 0 \) when \(-1 < x < 1\) and \( \lim_{n \to \infty} x^n = \infty \) when \( x > 1. \)

22. \( \lim_{n \to \infty} x^{1/n} = 1 \) for \( x \in \mathbb{R}. \)

### CONTINUITY

1. **Continuity:** Let \( A \subseteq \mathbb{R} \) and \( f: A \to \mathbb{R} \) be a given function and \( a \in \mathbb{R}. \) For each, given \( \varepsilon > 0, \) then there exists a \( \delta > 0 \) such that \( 0 < |x - a| < \delta \Rightarrow |f(x) - f(a)| < \varepsilon, \) then \( f(x) \) is called “Continuous function” at the point ‘a’ (or) continuity at the point ‘a’. It is denoted by \( \lim_{x \to a} f(x) = f(a) \) i.e., if limit = value (Left limit = Right limit = Value), then \( f(x) \) is continuous function.

2. **Discontinuous function:** If \( \lim_{x \to a} f(x) \neq f(a) \) (Limit \( \neq \) Value), then \( f(x) \) is called “Discontinuous function” at the point ‘a’.

3. **Types of Discontinuity:**
   - **Removable Discontinuity:**
     \( \lim_{x \to a} f(x) \neq f(a) \) (i.e., Limit \( \neq \) Value)
   - **Discontinuity of first kind (or) Jump Discontinuity:**
     \( \lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x) \) (i.e., Left limit \( \neq \) Right limit)
   - **Discontinuity of second kind:** Neither Left limit nor Right limit exist.
   - **Mixed Discontinuity:** \( \lim_{x \to a^+} f(x) \) does not exist; \( \lim_{x \to a^-} f(x) \) exist and it is may be equal or may not be equal to value. (or) \( \lim_{x \to a^+} f(x) \) does not exist; \( \lim_{x \to a^-} f(x) \) exist and it is may be equal or may not be equal to value.

4. If \( f \) is continuous at every point in a set \( A, \) then we say that \( f \) is continuous in the set \( A. \)

5. \( f(x) \) is continuous in \( [a, b] \Rightarrow f(x) \) is right continuous at \( x = a; \) \( f(x) \) is continuous in \((a,b)\) and \( f(x) \) is left continuous at \( x = b. \)

6. The constant function is continuous on \( \mathbb{R}. \)

7. The identity function \( f(x) = x \) is continuous on \( \mathbb{R}. \)

8. \( f(x) = x^n, n \in \mathbb{N} \) is continuous on \( \mathbb{R}. \)

9. The polynomial function \( f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n \) is continuous on \( \mathbb{R}. \)
10. $f(x) = x^n, \ x > 0$ and $n \in \mathbb{R}$ is continuous on $\mathbb{R}^+$.  \\
11. $f(x) = e^x$ is continuous on $\mathbb{R}$.  \\
12. $f(x) = \log x$ is continuous on $\mathbb{R}^+$.  \\
13. $f(x) = \log |x|$ is continuous on $\mathbb{R} - \{0\}$.  \\
14. $f(x) = \sin x, \ f(x) = \cos x$ are continuous on $\mathbb{R}$.  \\
15. $f(x) = \tan x$ is continuous on $\mathbb{R} - \{(2n + 1)\frac{\pi}{2}, \ n \in \mathbb{Z}\}$  \\
16. $f(x) = \cot x$ is continuous on $\mathbb{R} - \{(n\pi, \ n \in \mathbb{Z}\}$  \\
17. $f(x) = \cosec x$ is continuous on $\mathbb{R} - \{(n\pi, \ n \in \mathbb{Z}\}$  \\
18. $f(x) = \sec x$ is continuous on $\mathbb{R} - \{(2n + \frac{\pi}{2}, \ n \in \mathbb{Z}\}$  \\
19. $f(x) = [x]$ is discontinuous at integer points ($\mathbb{Z}$) and continuous at non-integer points ($\mathbb{R} - \mathbb{Z}$).  \\
20. If $f(x)$ and $g(x)$ are continuous at $x = a$ then  \\
21. a) $f(x) \pm g(x)$ is continuous at $x = a$.  \\
22. b) $cf(x)$ is continuous at $x = a$ where $c$ is a real number.  \\
23. c) $f(x) \cdot g(x)$ is continuous at $x = a$.  \\
24. d) $f(x) / g(x)$ is continuous at $x = a$ provided $g(a) \neq 0$.  \\
25. If $f(x)$ is continuous at $x = a$ then $|f(x)|$ is also continuous at $x = a$. The converse need not be true.

 GTA DIFFERENTIATION

1. Let $f$ be a function defined on neighborhood of a real number $a$. Then $f$ is said to be differentiable or derivable at $a$ if  \\
2. $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists. The limit is called the derivative or differential coefficient of $f$ at $a$. It is denoted by $f'(a)$  \\
3. If $f$ is differentiable at $a$, then  \\
4. $f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$  \\
5. Let $f$ be a function defined on a neighborhood of a real number $a$. Then $f$ is said to be right differentiable at $a$ if  \\
6. $\lim_{x \to a^+} \frac{f(x) - f(a)}{x - a}$ exists. The limit is called the right derivative of $f$ at $a$. It is denoted by $f'(a^+)$  \\
7. Similarly the left derivative of a function $f$ at $a$ is defined as  \\
8. $f'(a^-) = \lim_{x \to a^-} \frac{f(x) - f(a)}{x - a}$  \\
9. Let $f$ be a function defined on a neighborhood of a real number $a$. Then $f$ is differential at $a$ if  \\
10. $f(a^-) = f'(a^-)$  \\
11. $f'(a^+)$.
6. Let \( f \) be a function defined on \([a, b]\). Then \( f \) is differentiable at \( x \in (a, b) \) if
   i) \( f \) is differentiable at \( C \) where \( C \in (a, b) \)
   ii) \( f \) is right differentiable at \( a \)
   iii) \( f \) is left differentiable at \( b \)

7. Let \( f \) be a function defined on \([a, b]\). Then \( f \) is differentiable at \( x \in (a, b) \) if
   \[
   \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = L
   \]
   or Equivalent Form: \( f \) is differentiable at \( a \) if
   \[
   \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = L
   \]
   If \( y = f(x) \) is the given function then
   \( f'(x) \) is denoted by \( l + \frac{N/2-m}{f} \times C \), or \( D Y \) or \( y'_1 \) or \( y_1 \).

8. If a function \( f \) is differentiable at \( a \), then \( f \) is continuous at \( a \).

9. If a function \( f \) is discontinuous at \( a \) then \( f \) need not be differentiable at \( a \).

10. \( \frac{d}{dx}(x^n) = nx^{n-1} \)

11. \( \frac{d}{dx}(x) = 1 \)

12. \( \frac{d}{dx}(\cos \tan t) = 0 \)

13. \( \frac{d}{dx}(e^x) = e^x \)

14. \( \frac{d}{dx}(\log x) = \frac{1}{x} \)

15. \( \frac{d}{dx}(a^x) = a^x \log a \)

16. \( \frac{d}{dx}(\sin x) = \cos x \)

17. \( \frac{d}{dx}(\cos x) = -\sin x \)

18. \( \frac{d}{dx}(\tan x) = \sec^2 x \)

19. \( \frac{d}{dx}(\cot x) = -\csc^2 x \)

20. \( \frac{d}{dx}(\sec x) = \sec x \tan x \)

21. \( \frac{d}{dx}(\csc x) = -\csc x \cot x \)

22. \( \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \)

23. \( \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}} \)

24. \( \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \)

25. \( \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2} \)

26. \( \frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}} \)

27. \( \frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}} \)

28. \( \frac{d}{dx}(\sinh x) = \cosh x \)

29. \( \frac{d}{dx}(\cosh x) = \sinh x \)

30. \( \frac{d}{dx}(\tanh x) = \sec^2 h x \)

31. \( \frac{d}{dx}(\coth x) = -\csc^2 h x \)

32. \( \frac{d}{dx}(\sec h x) = \sec h x \tan h x \)

33. \( \frac{d}{dx}(\csc h x) = -\csc h x \cot h x \)
34. \( \frac{d}{dx} (\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}} \)

35. \( \frac{d}{dx} (\cosh^{-1} x) = \frac{1}{x^2 - 1} \)

36. \( \frac{d}{dx} (\tanh^{-1} x) = \frac{1}{\sqrt{1-x^2}} \)

37. \( \frac{d}{dx} (\coth^{-1} x) = \frac{1}{\sqrt{1-x^2}} \)

38. \( \frac{d}{dx} (\sech^{-1} x) = \frac{-1}{x\sqrt{1-x^2}} \)

39. \( \frac{d}{dx} (\text{coth}^{-1} x) = \frac{-1}{x\sqrt{1+x^2}} \)

\[ (iii) \quad g'(x) \neq 0, \forall x \in (a, b) \text{ then there exists } c \in (a, b) \text{ such that } \frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}. \]

\[ \text{MAXIMA AND MINIMA} \]

Definition:-
Let \( f : A \to \mathbb{R} \) be a function. Then
1. \( f \) is said to be Monotonically Increasing on \( A \) if \( x_1, x_2 \in A, \ x_1 < x_2 \implies f(x_1) \leq f(x_2) \)
2. \( f \) is said to be Strictly Increasing on \( A \) if \( x_1, x_2 \in A, x_1 < x_2 \implies f(x_1) < f(x_2) \)
3. \( f \) is said to be Monotonically Decreasing on \( A \) if \( x_1, x_2 \in A, x_1 < x_2 \implies f(x_1) \geq f(x_2) \)
4. \( f \) is said to be Strictly Decreasing on \( A \) if \( x_1, x_2 \in A, x_1 < x_2 \implies f(x_1) > f(x_2) \)
5. \( f \) is said to be Monotonic on \( A \) if \( f \) is either monotonically increasing or monotonically decreasing on \( A \).

Maxima & Minima for one variable function:-
Let \( y = f(x) \) be a given function.

Working Rule:
1. Find \( \frac{dy}{dx} \)
2. Let \( \frac{dy}{dx} = 0 \) and solve this equation for \( x \) values.
Let the values are a, b, c, ..........These points are called Critical Points.

3. Find \( \frac{d^2 y}{dx^2} \)

4. (i) If \( \frac{d^2 y}{dx^2} < 0 \) at \( x = a \), then \( f(x) \) is maximum at the point \( x = a \)
\( \therefore \) Max. value = \( f(a) \)

(ii) If \( \frac{d^2 y}{dx^2} > 0 \) at \( x = a \), then \( f(x) \) is maximum at the point \( x = a \)
\( \therefore \) Min. value = \( f(a) \)

(iii) If \( \frac{d^2 y}{dx^2} = 0 \) at \( x = a \), then \( f(x) \) is neither max. nor min.
\( \therefore f(x) \) is point of inflection at the point \( x = a \)

Maxima & Minima for two variable function: Let \( y = f(x, y) \) be a given function.

Working Rule:-

1. Find \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) values.

2. Let \( \frac{\partial f}{\partial x} = 0 \) and \( \frac{\partial f}{\partial y} = 0 \) and solve these for \( x \) and \( y \) values.

Let the points are \( (a_1, b_1), (a_2, b_2), \ldots \)
These points are called “Stationary Points”.

3. Find \( 1 = \frac{\partial^2 f}{dx^2}, \quad m = \frac{\partial^2 f}{\partial x \partial y} \) and \( n = \frac{\partial^2 f}{\partial y^2} \) values.

4. (i) If \( ln - m^2 > 0 \) and \( l < 0 \) at \( (a_1, b_1) \),
then \( f(x, y) \) is maximum at \( (a_1, b_1) \).
\( \Rightarrow \) Max. value = \( f(a_1, b_1) \)

(ii) If \( ln - m^2 > 0 \) and \( l > 0 \) at \( (a_1, b_1) \),
then \( f(x, y) \) is minimum at \( (a_1, b_1) \).

(iii) If \( ln - m^2 < 0 \) and \( l > 0 \) at \( (a_1, b_1) \),
then neither max. nor min.
\( \therefore \) The point \( (a_1, b_1) \) is called a “Saddle point” and \( f(x, y) \) has a point of inflexion at \( (a_1, b_1) \).

(iv) If \( ln - m^2 = 0 \) at \( (a_1, b_1) \), then we can’t discuss about max. and min.

PARTIAL DIFFERENTIATION:

Let \( z = f(x, y) \) be a function of two variables \( x \) and \( y \). Then
\[ \lim_{x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}, \] if it exists, is said to be partial derivative or partial differential coefficient of \( z \) or \( f(x, y) \), w.r.t. \( x \). It is denoted by the symbol \( \frac{\partial f}{\partial x} \) or \( \frac{\partial z}{\partial x} \) or \( f_1 \).

Thus we see that for the partial derivative of \( z = f(x, y) \) with respect to \( x, y \) is kept constant.
Similarly, the partial derivative of \( z = f(x, y) \) w.r.t. ‘y’ keeping ‘x’ as constant is defined as 
\[
\lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}
\]
and is denoted by \( \frac{\partial z}{\partial x} \) or \( \frac{\partial f}{\partial y} \).

**HIGHER ORDER PARTIAL DERIVATIVES:**
In general the first order partial derivatives \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) are also functions of \( x \) and \( y \) and they can be differentiated repeatedly to get higher order partial derivatives.

\[
\frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right) = \frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial y} \right) = \frac{\partial^2 f}{\partial y^2},
\]

So 
\[
\frac{\partial}{\partial x} \left( \frac{\partial^2 f}{\partial x \partial y} \right) = \frac{\partial^3 f}{\partial x^2 \partial y}, \quad \frac{\partial}{\partial y} \left( \frac{\partial^2 f}{\partial x \partial y} \right) = \frac{\partial^3 f}{\partial x \partial y^2},
\]

\[
\frac{\partial}{\partial x} \left( \frac{\partial^2 f}{\partial x^2} \right) = \frac{\partial^3 f}{\partial x^3}, \quad \frac{\partial}{\partial y} \left( \frac{\partial^2 f}{\partial y^2} \right) = \frac{\partial^3 f}{\partial y^3},
\]

\[
\frac{\partial}{\partial x} \left( \frac{\partial^2 f}{\partial x^2 \partial y} \right) = \frac{\partial^3 f}{\partial x^2 \partial y^2}, \quad \frac{\partial}{\partial y} \left( \frac{\partial^2 f}{\partial x^2 \partial y} \right) = \frac{\partial^3 f}{\partial x \partial y^3},
\]

\[
\frac{\partial}{\partial y} \left( \frac{\partial^2 f}{\partial y^2 \partial x^2} \right) = \frac{\partial^3 f}{\partial y^2 \partial x^2} \text{ and so on.}
\]

**THE CHAIN RULE OF PARTIAL DIFFERENTIATION:**
(i) If \( z = f(x, y) \) where \( x = \phi(t) \), \( y = \psi(t) \) then \( z \) is called a composite function of a variable \( t \).
(ii) If \( z = f(x, y) \) where \( x = \phi(u, v) \), \( y = \psi(u, v) \) then \( z \) is called a composite function of two variables \( u \) and \( v \).

**Total differential coefficient:**
In the differential form, this result can be written as 
\[ du = \frac{\partial u}{\partial x} \cdot dx + \frac{\partial u}{\partial y} \cdot dy \]
\( du \) is called the total differential of \( u \).

**Jacobian:** Let \( u = u(x, y) \) and \( v = v(x, y) \) are two given functions. The Jacobian of \( u, v \) with respect to \( x, y \) is denoted by 
\[
J \left[ \frac{u, v}{x, y} \right] \text{ or } \frac{\partial (u, v)}{\partial (x, y)} \Rightarrow 
\]
\[
J \left[ \frac{u, v}{x, y} \right] = \begin{vmatrix}
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}
\end{vmatrix}
\]

**Functionally dependent and independent**
Functionally Dependent : \( J(u,v)=0 \)
Functionally Independent : \( J(u,v)\neq0 \)

**Note:** Functionally dependent is also known as Linearly dependent.

**INDIFFERENT INTEGRATION**

**Formulas:**
1. If \( n \neq -1 \) then 
\[
\int x^n \, dx = \frac{x^{n+1}}{n+1} + c
\]
<table>
<thead>
<tr>
<th>No.</th>
<th>Integral</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>( \int dx = x + c )</td>
<td>( \int \frac{1}{\sqrt{x}} )</td>
</tr>
<tr>
<td>3.</td>
<td>( \int \frac{1}{x} )</td>
<td>( \log</td>
</tr>
<tr>
<td>4.</td>
<td>( \int e^x )</td>
<td>( e^x + c )</td>
</tr>
<tr>
<td>5.</td>
<td>( a &gt; 0, a \neq 1 ) then ( \int a^x dx = \frac{a^x}{\log a} + c )</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>( \int \cos x )</td>
<td>( \sin x + c )</td>
</tr>
<tr>
<td>7.</td>
<td>( \int \sin x )</td>
<td>( -\cos x + c )</td>
</tr>
<tr>
<td>8.</td>
<td>( \int \sec^2 x )</td>
<td>( \tan x + c )</td>
</tr>
<tr>
<td>9.</td>
<td>( \int \csc^2 x )</td>
<td>( -\cot x + c )</td>
</tr>
<tr>
<td>10.</td>
<td>( \int \sec x \tan x )</td>
<td>( \sec x + c )</td>
</tr>
<tr>
<td>11.</td>
<td>( \int \cos ecx \cot x )</td>
<td>( -\cos ecx + c )</td>
</tr>
<tr>
<td>12.</td>
<td>( \int \csc ecx \cot x )</td>
<td>( -\cot ecx + c )</td>
</tr>
<tr>
<td>13.</td>
<td>( \int \frac{1}{\sqrt{1-x^2}} )</td>
<td>( \sin^{-1} x + c = -\cos^{-1} x + c )</td>
</tr>
<tr>
<td>14.</td>
<td>( \int \frac{1}{1+x^2} )</td>
<td>( \tan^{-1} x + c = -\cot^{-1} x + c )</td>
</tr>
<tr>
<td>15.</td>
<td>If ( x &gt; 1 ), then ( \int \frac{1}{x\sqrt{x^2-1}} )</td>
<td>( \sec^{-1} x + c = -\csc ec^{-1} x + c )</td>
</tr>
<tr>
<td>16.</td>
<td>and if ( x &lt; -1 ) then ( \int \frac{1}{x\sqrt{x^2-1}} )</td>
<td>( -\sec^{-1} x + c = \csc ec^{-1} x + c )</td>
</tr>
<tr>
<td>17.</td>
<td>( \int \sinh x )</td>
<td>( \cosh x + c )</td>
</tr>
<tr>
<td>18.</td>
<td>( \int \cosh x )</td>
<td>( \sinh x + c )</td>
</tr>
<tr>
<td>19.</td>
<td>( \int \sec h^2 x )</td>
<td>( \tanh x + c )</td>
</tr>
<tr>
<td>20.</td>
<td>( \int \sec h \tanh x )</td>
<td>( \sec h x + c )</td>
</tr>
<tr>
<td>21.</td>
<td>( \int \cos ech )</td>
<td>( \cosh x + c )</td>
</tr>
<tr>
<td>22.</td>
<td>( \int \frac{1}{\sqrt{1+x^2}} )</td>
<td>( \sinh^{-1} x + c )</td>
</tr>
<tr>
<td>23.</td>
<td>( \int \frac{1}{\sqrt{x^2-1}} )</td>
<td>( \cosh^{-1} x + c )</td>
</tr>
<tr>
<td>24.</td>
<td>If ( f(x) ) is an integrable function and ‘k’ is a real number, then ( \int (kf)(x)dx = k\int f(x)dx )</td>
<td></td>
</tr>
<tr>
<td>25.</td>
<td>( f(x), g(x) ) are two integrable functions, then ( \int (f+g)(x)dx = \int f(x)dx + \int g(x)dx )</td>
<td></td>
</tr>
<tr>
<td>26.</td>
<td>If ( f(x), g(x) ) are two integrable functions, then ( \int (f-g)(x)dx = \int f(x)dx - \int g(x)dx )</td>
<td></td>
</tr>
<tr>
<td>27.</td>
<td>If ( f_1(x), f_2(x), \ldots, f_n(x) ) are integrable functions, then ( \int (f_1+f_2+\ldots+f_n)(x)dx )</td>
<td></td>
</tr>
<tr>
<td>28.</td>
<td>If ( f(x), g(x) ) are two integrable functions and ( k, l ) are two real numbers, then ( \int (kf+lg)(x)dx = k\int f(x)dx + l\int g(x)dx )</td>
<td></td>
</tr>
<tr>
<td>29.</td>
<td>If ( \int f(x)dx = g(x) ) then ( \int f(ax+b)dx = \frac{1}{a}g(ax+b)+c )</td>
<td></td>
</tr>
</tbody>
</table>
30. If \( f(x) \) is a differentiable function, then
\[ \int \frac{f'(x)}{f(x)} \, dx = \log |f(x)| + c \]
31. \( \int \tan x \, dx = \log |\sec x| + c \)
32. \( \int \cot x \, dx = \log |\sin x| + c \)
33. \( \int \sec x \, dx = \log |\sec x + \tan x| + c \)
\[ = \log |\tan(\pi/4 + x/2)| + c \]
34. \( \int \csc x \, dx = \log |\csc x - \cot x| + c \)
\[ = \log |\tan x/2| + c \]
35. If \( f(x) \) is a differentiable function and \( n \neq -1 \), then
\[ \int \left[ f(x) \right]^{n} f'(x) \, dx = \frac{\left[ f(x) \right]^{n+1}}{n+1} + c \]
36. If \( n = -1 \) then
\[ \int \frac{f'(x)}{f(x)} \, dx = \log |f(x)| + c \]
37. \( \int \frac{f'(x)}{\sqrt{f(x)}} \, dx = 2 \sqrt{f(x)} + c \)
38. If \( \int f(x) \, dx = F(x) \) and \( g(x) \) is a differentiable function, then
\[ \int (f(g(x)))g'(x) \, dx = F\left[ g(x) \right] + c \]
39. \( \int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \left( \frac{x}{a} \right) + c \)
40. \( \int \frac{1}{\sqrt{a^2 + x^2}} \, dx = \sinh^{-1} \left( \frac{x}{a} \right) + c \)
41. \( \int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \cosh^{-1} \left( \frac{x}{a} \right) + c \)
42. \( \int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + c \)
43. \( \int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \log \left| \frac{a + x}{a - x} \right| + c \)
44. \( \int \frac{1}{x^2 - a^2} \, dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c \)
45. \( \int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) + c \)
46. \( \int \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} + \frac{a^2}{2} \sinh^{-1} \left( \frac{x}{a} \right) + c \)
47. \( \int x \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1} \left( \frac{x}{a} \right) + c \)
48. If the given integral is of the form
\[ \int \frac{px + q}{ax^2 + bx + c} \, dx \]
then take
\[ px + q = A \frac{dx}{(ax^2 + bx + c)} + B \]
49. If the given integral is of the form
\[ \int \sqrt{(px + q)(ax^2 + bx + c)} \, dx \]
then take
\[ px + q = A \frac{dx}{(ax^2 + bx + c)} + B \]
50. If the given integral is of the form
\[ \int \frac{1}{(px + q)\sqrt{ax^2 + bx + c}} \, dx \]
then put
\[ px + q = \frac{1}{t} \]
51. If the given integral is of the form
\[ \int \frac{1}{(ax^2 + b)\sqrt{cx^2 + d}} \, dx \]
then put \( x = \frac{1}{t} \)
52. If the given integral is of the form
\[ \int \frac{px + q}{\sqrt{ax + b}} \, dx \]
or
\[ \int \sqrt{ax + b} \, dx \]
or
\[
\int (px + q) \sqrt{ax + b} \, dx \quad \text{or} \quad \int \frac{1}{(px + q) \sqrt{ax + b}} \, dx \text{ the put } ax + b = t^2
\]

and hence \( dx = \frac{1}{a} - 2t \, dt \)

53. If the integral is of the form
\[
\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} \quad \text{or} \quad \int \frac{dx}{a \cos^2 x + b \sin x \cos x + c \sin^2 x}
\]

then multiply both numerator and denominator with \( \sec^2 x \) and take \( \tan x = t \)

54. If the integral is of the form
\[
\int \frac{dx}{a + b \cos x} \quad \text{or} \quad \int \frac{dx}{a + b \sin x} \quad \text{or} \quad \int \frac{dx}{a \cos x + b \sin x + c}
\]

take \( \tan \frac{x}{2} = t \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} \, dx = dt \)

\[
(1 + \tan^2 x / 2) \, dx = 2dt \quad \Rightarrow (1 + t^2) \, dx = 2dt
\]

\[
\Rightarrow dx = \frac{2dt}{1 + t^2}
\]

\[
\sin x = \frac{2 \tan x / 2}{1 + \tan^2 x / 2} = \frac{2t}{1 + t^2},
\]

\[
\cos x = \frac{1 - \tan^2 x / 2}{1 + \tan^2 x / 2} = \frac{1 - t^2}{1 + t^2}
\]

55. If the integral is of the form
\[
\int \frac{a \cos x + b \sin x}{a \cos x + d \sin x} \, dx, \text{ take } a \cos x + b \sin x = \frac{e^{ax} \cos bx}{a^2 + b^2} (a \cos bx + b \sin bx) + c
\]

56. Integration by parts: If \( f(x) \) and \( g(x) \) are two integrable functions then
\[
\int f(x) g(x) \, dx = f(x) \int g(x) \, dx - \int f'(x) \int [g(x)] \, dx \, dx
\]

57. If \( u \) and \( v \) are two functions of \( x \) then
\[
\int u \, dv = uv - \int v \, du.
\]

58. If \( u \) and \( v \) are two functions of \( x; u', u'', u''' ..., \) denote the successive derivatives of \( u \) and \( v_1, v_2, v_3, ... \). Denote the successive integrals of \( v \) then the extension of Integration by parts is
\[
\int u \, v \, dx = u_1 v_1 - u_2 v_2 + u_3 v_3 - u_4 v_4 + ......\]

59. In integration by parts, the first function will be taken as in the following order.
Inverse functions, logarithmic functions, Algebraic functions, Trigonometric functions and exponential functions. (To remember this a phrase ILATE)

60. \[
\int e^{ax} \cos bx \, dx = \frac{e^{ax} \cos bx}{a^2 + b^2} (a \cos bx + b \sin bx) + c
\]
61. \[ \int e^{ax} \sin bxdx = \frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx) \]

62. \[ \int e^x \left[ f(x) + f'(x) \right] dx = e^x f(x) + c \]

63. \[ \int e^{-x} \left[ f(x) - f'(x) \right] dx = -e^{-x} f(x) + c \]

64. If \[ I_n = \int x^n e^{ax} dx \] then \[ I_n = e^{ax} - \frac{n}{a} I_{n-1} \] where \( n \) is a positive integer.

65. \[ I_n I_n = \int \sin^n x dx \] then 
\[ I_n = -\sin^{n-1} x \cos x + \frac{n-1}{2} I_{n-2} \] where \( n \) is a positive integer.

66. If \[ I_n = \int \cos^n x dx \] then 
\[ I_n = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{2} I_{n-2} \]

67. If \[ I_n = \int \cos^n x dx \] then 
\[ I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2} \]

68. If \[ I_n = \int \tan^n x dx \] then 
\[ I_n = -\cot^{n-1} x \frac{1}{n-1} - I_{n-2} \]

69. If \[ I_n = \int \sec^n x dx \] then 
\[ I_n = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} I_{n-2} \]

70. If \[ I_n = \int \csc^n x dx \] then 
\[ I_n = -\cos e^{n-2} x \cot x + \frac{n-2}{n-1} I_{n-2} \]

71. If \[ I_n = \int (\log x)^n dx \] then 
\[ I_n = x(\log x)^n - nI_{n-1} \]

72. If \[ I_{m,n} = \int \sin^m x \cos^n x dx \] 
\[ I_{m,n} = \frac{\sin^{m-1} x \cos^{n-1} x}{m + n} + \frac{n-1}{m + n} I_{m-2n} \]

\[ \frac{\sin^{m-1} x \cos^{n-1} x}{m + n} + \frac{n-1}{m + n} I_{m-2n} \]

**Fourier Series**

**INTRODUCTION:**

- Suppose that a given function \( f(x) \) defined on \([-\pi, \pi]\) or \([0, 2\pi]\) or in any other interval can be expressed as a trigonometric series as

\[ f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + ... + a_n \cos nx + b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + ... + b_n \sin nx + ... \]

- Where the \( a \)'s and \( b \)'s are constants within a desired range of values of the variable. Such series is known as the Fourier series for \( f(x) \) and the constants \( a_0, a_n, b_n (n = 1, 2, 3 ... ) \) are called Fourier coefficients of \( f(x) \).
PERIODIC FUNCTIONS:
A function \( f(x) \) is said to be of period \( T \) or to be periodic with period \( T > 0 \) if for all \( x \), \( f(x + T) = f(x) \), and \( T \) is the least of such values.

EULER’S FORMULAE:
The Fourier Series for the function \( f(x) \) in the interval \( C \leq x \leq C + 2\pi \) is given by

\[
f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)
\]

where

\[
a_0 = \frac{1}{\pi} \int_{C}^{C+2\pi} f(x) \, dx
\]

\[
a_n = \frac{1}{\pi} \int_{C}^{C+2\pi} f(x) \cos nx \, dx
\]

\[
b_n = \frac{1}{\pi} \int_{C}^{C+2\pi} f(x) \sin nx \, dx
\]

These values of \( a_0, a_n, b_n \) are known as Euler’s formulae.

Case 1: If \( f(x) \) is to be expanded as a Fourier series in the interval \( 0 \leq x \leq 2\pi \), put \( C = 0 \), then the formulae (A) reduces to

\[
a_0 = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \, dx
\]

\[
a_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \cos nx \, dx
\]

\[
b_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \sin nx \, dx
\]

Case 2: If \( f(x) \) is to be expanded as a Fourier series in \( [-\pi, \pi] \), put \( C = -\pi \); the interval becomes \( -\pi \leq x \leq \pi \) and the formulae (A) reduces to

\[
a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \, dx
\]

\[
a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx
\]

\[
b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx
\]
This $\phi (x, y, z)$ is called a scalar point function defined on the region. Similarly if to each point $P(x, y, z)$ we associate a unique vector $\overrightarrow{f}(x, y, z), \overrightarrow{f}$ is called a vector point function.

- **Vector Differential Operator:**
  - **Definition:** The vector differential operator $\nabla$ (read as del) is defined as $\nabla \equiv \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$. This operator possesses properties analogous to those of ordinary vectors as well as differential operator. We will define now some quantities known as “gradient”, “divergence” and “curl” involving this operator $\nabla$. We must note that this operator has no meaning by itself unless it operates on some function suitably.

- **Gradient of a Scalar Point Function:**
  - Let $\phi(x, y, z)$ be a scalar point function of position defined in some region of space.
  - Then the vector function $\overrightarrow{\nabla} \phi = \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}$ is known as the gradient of $\phi$ and is denoted by $\text{grad } \phi$ or $\nabla \phi$.
  - $\nabla \phi = \left( \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z} \right) \phi = \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}$

- **Properties:**
  1. If $f$ and $g$ are two scalar functions then $\text{grad } (f \pm g) = \text{grad } f \pm \text{grad } g$
  2. The necessary and sufficient condition for a scalar point function to be constant is that $\nabla f = \vec{0}$
  3. $\text{Grad } (fg) = f(\text{grad } g) + g(\text{grad } f)$
  4. If $c$ is a constant, $\text{grad } (cf) = c(\text{grad } f)$
  5. $\text{grad } \left( \frac{f}{g} \right) = \frac{g(\text{grad } f) - f(\text{grad } g)}{g^2}, (g \neq 0)$
  6. Let $\overrightarrow{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $d\overrightarrow{r} = (dx)\hat{i} + (dy)\hat{j} + (dz)\hat{k}$. If $\phi$ is any scalar point function then $d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz = \left( \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z} \right) \left( \hat{i} dx + \hat{j} dy + \hat{k} dz \right) = \nabla \phi \cdot d\overrightarrow{r}$

- **Directional Derivative:**
  - Let $\phi(x, y, z)$ be a scalar function defined throughout some region of space. Let this function have a value $\phi$ at a point $P$ whose position vector referred to the origin $O$ is $\overrightarrow{OP} = \overrightarrow{r}$. Let $\phi + \Delta \phi$ be the value of the function at neighbouring point $Q$. If $\overrightarrow{OQ} = \Delta \overrightarrow{r}$ then $\overrightarrow{PQ} = \Delta \overrightarrow{r}$.
  - Let $\Delta r$ be the length of $\Delta \overrightarrow{r}$.
  - $\frac{\Delta \phi}{\Delta r}$ gives a measure of the rate at which $\phi$ change when we move from $P$ to $Q$. The limiting value of $\frac{\Delta \phi}{\Delta r}$ as $\Delta r \to 0$ is called the derivative of $\phi$ in the direction of $\overrightarrow{PQ}$. 

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or simply directional derivative of $\phi$ at $P$ and is denoted by $\frac{d\phi}{dr}$.

**Note:** The directional derivative of a scalar point function $\phi$ at a point $P(x, y, z)$ in the direction of a unit vector $\mathbf{e}$ is equal to $\mathbf{e}.\nabla\phi$.

---

**Differential Equations**

- **Order and Degree of a Differential Equation:-**
  - **Order:** The order of the highest order derivative.
  - **Degree:** The Degree of the highest order derivative, when it is free from square root, cube root and decimal values etc.

- **First Order and First Degree D.E.:**
  \[
  \frac{dy}{dx} = f(x, y), \text{ where } f(x, y) \text{ be a function of } x \text{ and } y.
  \]

- **Methods to solve first order and first degree D.E.:**
  - **Method-1:** Method of variable and separable.
    \[
    \frac{dy}{dx} = \frac{f(x)}{g(y)} \Rightarrow f(x)dx = g(y)dy, \text{ Integrating on both sides } \int f(x)dx - \int g(y)dy + c
    \]
    \[
    \Rightarrow \int f(x)dx - \int g(y)dy = c, \text{ which is a required solution. This is called Method of Variable and Separable.}
    \]

- **Method-2:** Homogeneous Differential Equation (H.D.E.)
  - **Homogeneous Function:** Let $f(x, y)$ be a given function. If there exists a real number $k$ such that $f(kx, ky) = k^nf(x, y)$, then $f(x, y)$ is called Homogeneous Function, where '$n$' is the Degree of Homogeneous function.
  - The first order and first degree differential equation $\frac{dy}{dx} = f(x, y)$, where $f(x, y)$ be a Homogeneous function of degree 0 is called Homogeneous Differential Equation.
  - **Note:**
    - If $\frac{dy}{dx} = f(x, y)$ be a H.D.E. then sub. $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$
    - If $\frac{dy}{dx} = f(x, y)$ be a H.D.E. then sub. $x = vy \Rightarrow \frac{dy}{dx} = v + y \frac{dv}{dy}$

- **Method-3:** Non Homogeneous Differential Equation (N.H.D.E.)
  \[
  \frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}, \text{ where } a_1, b_1, c_1; a_2, b_2, c_2 \text{ are Real Numbers and at least one constant } c_1 \text{ and } c_2 \text{ is Non-zero.}
  \]

- **Method-4:** Exact Differential Equation (E.D.E.)
  \[
  M(x, y)dx + N(x, y)dy = 0
  \]
  (or)
Mdx + Ndy = 0 be a first order and first degree D.E.

**Condition:** Mdx + Ndy = 0 is E.D.E.

If and only if (iff) \( \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \)

The required General Solution is

\[
\int Mdx + \int Ndy = C \quad \text{(In terms of N, free from x)}
\]

**Integrating Factor (I.F.):** Integrating Factor reduces Non Exact Differential Equation into Exact Differential Equation.

**Methods of Integrating Factor:**

**Method – (i):** Method of Inspection:

Formulas:

1. \( d(xy) = xdy + ydx \)
2. \( d \left( \frac{x}{y} \right) = \frac{ydx - xdy}{y^2} \)
3. \( d \left( \frac{y}{x} \right) = \frac{xdy - ydx}{x^2} \)
4. \( d \left( \frac{e^x}{y} \right) = \frac{ye^x dx - e^x dy}{y^2} \)
5. \( d \left( \frac{e^y}{x} \right) = \frac{xe^y dy - e^y dx}{x^2} \)
6. \( d \left[ \tan^{-1} \left( \frac{x}{y} \right) \right] = \frac{ydx - xdy}{x^2 + y^2} \)
7. \( d \left[ \tan^{-1} \left( \frac{y}{x} \right) \right] = \frac{xdy - ydx}{x^2 + y^2} \)
8. \( d \left[ \log \left( \frac{x}{y} \right) \right] = \frac{ydx - xdy}{xy} \)
9. \( d \left[ \log \left( \frac{y}{x} \right) \right] = \frac{xdy - ydx}{xy} \)
10. \( d \left[ \frac{1}{2} \left( \log \left( x^2 + y^2 \right) \right) \right] = \frac{xdx + ydy}{x^2 + y^2} \)

**Method – (ii):** If Mdx + Ndy = 0 is a Homogeneous Differential Equation and Mx + Ny \( \neq 0 \), then Integrating Factor is \( \frac{1}{Mx + Ny} \).

**Method – (iii):** If the given Differential Equation is in the form \( f(x, y)dx + xg(x, y)dy = 0 \) and \( Mx - Ny \neq 0 \), then Integrating Factor is \( \frac{1}{Mx - Ny} \), where M = yf(x, y) and N = xg(x, y).

**Method – (iv):** If \( \frac{1}{M} \left( \frac{\partial N}{\partial y} - \frac{\partial M}{\partial x} \right) = f(y) \), function of y, then the Integrating Factor of Mdx + Ndy = 0 is \( e^{\int f(y)dy} \).

**Method – (v):** If \( \frac{1}{N} \left( \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = f(x) \), function of x, then the Integrating Factor of Mdx + Ndy = 0 is \( e^{\int f(x)dx} \).

**Method – (5):** Linear Differential Equation (L.D.E.):

**Linear Differential Equation in y:**

\[
\frac{dy}{dx} + p(x) y = Q(x) \quad \text{(or)} \quad \frac{dy}{dx} + p \cdot y = Q
\]

**General Solution:** \( ye^{\int pdx} = \int Qe^{\int pdx} dx + c \)
where $e^\int pdy$ is called I.F.

**Linear Differential Equation in x:**

$$\frac{dx}{dy} + p(y).x = Q(y) \quad \text{or} \quad \frac{dx}{dy} + p.x = Q$$

**General Solution:** $xe^{\int pdy} = \int Qe^{\int pdy}dy + c$

where $e^{\int pdy}$ is called I.F.

- **Bernoulli’s Equation:**
  - (i) **Bernoulli’s Equation in y:**
    $$\frac{dx}{dy} + p(x).y = Q(x)y^n \quad \text{where ‘}n\text{’ is a +ve integer.}$$
  - (ii) **Bernoulli’s Equation in x:**
    $$\frac{dx}{dy} + p(y).x = Q(y)x^n \quad \text{where ‘}n\text{’ is a +ve integer.}$$

**Note:** Reduce Bernoulli’s Equation into Linear Equation and find the required General Solution.

- **Applications to first order and first degree D.E.:**
  - **Orthogonal Trajectories (O.T.):**
    Two curves intersect mutually at a right angle.
  - **Types of O.T:**
    - **Cartesian Form (or) Cartesian Coordinates:**
      The given curve is in the form $f(x, y, c) = 0$, where ‘c’ is an arbitrary constant.

- **Linear Differential Equations of Second and Higher Order:**
  - **Definition:** An equation of the form
    $$\frac{d^n y}{dx^n} + P_1(x)\frac{d^{n-1} y}{dx^{n-1}} + P_2(x)\frac{d^{n-2} y}{dx^{n-2}} + \ldots + P_n(x)y = Q(x)$$
    \[\ldots\ldots\ldots (1)\]

  Where $P_1(x), P_2(x), \ldots, P_n(x)$ are Real constants and $Q(x)$ be a function of $x$ is called “Linear Differential equation of order “ $n$ or $n^{th}$ order Linear Differential Equation”.

We know that, $D = \frac{d}{dx}$

\[\therefore \frac{dy}{dx} = Dy, \quad \frac{d^2 y}{dx^2} = D^2 y, \quad \ldots, \quad \frac{d^n y}{dx^n} = D^n y\]
From equation (1), we get
\[ D^n y + P_1(x)D^{n-1} y + P_2(x)D^{n-2} y + \ldots + P_n(x) y = Q(x) \]
\[ \Rightarrow [D^n + P_1(x)D^{n-1} + P_2(x)D^{n-2} + \ldots + P_n(x)] y = Q(x) \]
\[ \Rightarrow [f(D)] y = Q(x) \quad \ldots \ldots \quad (2) \]

- If \( Q(x) = 0 \), then from equation (2), we get \([f(D)] y = 0\). This is called “nth order Linear Homogeneous Differential Equation” and the required general solution is \( y = y_c + y_p \)

Table for Complementary Function:

<table>
<thead>
<tr>
<th>S.No</th>
<th>Roots</th>
<th>Complimentary Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( m_1, m_2 ) are real and Distinct</td>
<td>( y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x} ), where ( c_1 &amp; c_2 ) are two arbitrary constants.</td>
</tr>
<tr>
<td>2</td>
<td>( m_1, m_2, m_3 ) are Real and Distinct</td>
<td>( y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} ) where ( c_1, c_2, c_3 ) are three arbitrary constants</td>
</tr>
<tr>
<td>3</td>
<td>( m, m ) are Real and equal</td>
<td>( y_c = (c_1 + c_2 x) e^{mx} )</td>
</tr>
<tr>
<td>4</td>
<td>( m, m, m ) are real and equal</td>
<td>( y_c = (c_1 + c_2 x + c_3 x^2) e^{mx} )</td>
</tr>
<tr>
<td>5</td>
<td>( m, m_1, m_1 )</td>
<td>( y_c = c_1 e^{mx} + (c_2 + c_3 x) e^{mx} )</td>
</tr>
<tr>
<td>6</td>
<td>( m_1, m_1, m )</td>
<td>( y_c = (c_1 + c_2 x) e^{m_1 x} + c_3 e^{mx} )</td>
</tr>
<tr>
<td>7</td>
<td>( \alpha \pm \beta i )</td>
<td>( y_c = e^{\alpha x} \left( c_1 \cos \beta x + c_2 \sin \beta x \right) )</td>
</tr>
<tr>
<td>8</td>
<td>( \alpha \pm \sqrt{\beta} )</td>
<td>( y_c = e^{\alpha x} \left( c_1 \cosh \sqrt{\beta} x + c_2 \sinh \sqrt{\beta} x \right) )</td>
</tr>
</tbody>
</table>

- **Particular Integral (yp):**

**Case (i):** \([f(D)] y = Q(x)\), where \( Q(x) = e^{ax} \), and ‘\( a \)’ is a constant.

For \( y_p \) value, sub. \( D=a \) in \( f(D) \)

**Note:**
1. If \( D - a = 0 \), then

\[
\frac{e^{ax}}{(D-a)^k} = \frac{x^k e^{ax}}{k!}
\]

2. \( \sinh x = \frac{e^x - e^{-x}}{2} \) and \( \cosh x = \frac{e^x + e^{-x}}{2} \)

**Case (ii):** \([f(D)] y = Q(x)\), where \( Q(x) = \sin ax \) (or) \( \cos ax \), and ‘\( a \)’ is a constant.
For \( y_p \) value sub. \( D^2 = -a^2 \) in \( f(D) \)

**Note:**

1. If \( D^2 + a^2 = 0 \), then
   \[ \frac{\sin ax}{D^2 + a^2} = -\frac{x}{2a} \cos ax \]
   \[ \frac{\cos ax}{D^2 + a^2} = \frac{x}{2a} \sin ax \]
2. \( D = \text{Differentiation and} \quad \frac{1}{D} = \text{Integration} \)
3. \( \sin A \cos B = \frac{\sin(A + B) + \sin(A - B)}{2} \)
4. \( \cos A \sin B = \frac{\sin(A + B) - \sin(A - B)}{2} \)
5. \( \cos A \cos B = \frac{\cos(A + B) + \cos(A - B)}{2} \)
6. \( \sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2} \)
7. \( \sin(-\theta) = -\sin \theta \) and \( \cos(-\theta) = \cos \theta \)
8. \( \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \) and \( \cos^2 \theta = \frac{1 + \cos 2\theta}{2} \)
9. \( \sin 3\theta = 3\sin \theta - 4\sin^3 \theta \)
10. \( \cos 3\theta = 4\cos^3 \theta - 3\cos \theta \)

**Case (iii):** \([f(D)]y = Q(x)\), where \( Q(x) = x^k \), and ‘k’ is a constant.

**Formulas:**

(i) \( (1 + x)^{-1} = 1 - x + x^2 - x^3 + \ldots \)
(ii) \( (1 - x)^{-1} = 1 + x + x^2 + x^3 + \ldots \)
(iii) \( (1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots \)
(iv) \( (1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots \)
(v) \( (1 + x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \ldots \)
(vi) \( (1 - x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \ldots \)

**Euler-Cauchy’s Equation:** An equation of the form

\[
x^n \frac{d^n y}{dx^n} + P_1(x) x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + P_2(x) x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \ldots + P_n(x) y = Q(x)
\]

is called “Euler-Cauchy’s Equation”.

From the above equation, we get

\[
x^n D^n y + P_1(x) x^{n-1} D^{n-1} y + P_2(x) x^{n-2} D^{n-2} y + \ldots + P_n(x) y = Q(x)
\]

\[
\Rightarrow [x^n D^n + P_1(x) x^{n-1} D^{n-1} + P_2(x) x^{n-2} D^{n-2} + \ldots + P_n(x)] y = Q(x) \quad \ldots \ldots \ldots (1)
\]

\[x D = \theta, \quad x^2 D^2 = \theta(\theta - 1), \quad x^3 D^3 = \theta(\theta - 1)(\theta - 2), \quad \ldots \ldots \ldots \]

Substitute all of these values in equation (1) and again simplifying,

We get the required general solution.

**Legendre’s Equation:**

\[
(ax + b)^n \frac{d^n y}{dx^n} + P_1(x)(ax + b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \ldots + P_n(x) y = Q(x)
\]
From the above equation, we get
\[(ax + b)^n D^n y + P_1(x)(ax + b)^{n-1} D^{n-1} y + P_2(x)(ax + b)^{n-2} D^{n-2} y + \ldots + P_n(x) y = Q(x)\]

\[\Rightarrow + P_2(x)(ax + b)^{n-2} D^{n-2} y + \ldots + P_n(x) y = Q(x)\]

Put \(ax + b = e^z \Rightarrow z = \log(ax + b)\)

\[(ax + b)^n D^n = a\theta, \quad (ax + b)^{n+2} D^{n+2} = a^2 (\theta)(\theta-1), \]

\[(ax + b)^3 D^3 = a^3 (\theta)(\theta-1)(\theta-2) \ldots \]

Substitute all of these values in equation (1) and again simplifying, we get the required solution.

### Method of Variation of Parameters:

\[
\frac{d^2 y}{dx^2} + P(x) \frac{dy}{dx} + Q(x) y = R(x)
\]

The required General Solution is \(y = y_e + y_p\)

Where \(y_p = Au + Bv\)

Where \(u\) and \(v\) are functions of Complementary function.

\[
A = - \int \frac{vR}{udv - vdu} \frac{dx}{dx} dx
\]

and \(B = \int \frac{uR}{udv - vdu} \frac{dx}{dx} dx\)

### Functions of Complex Variable:

- **Suppose** \(D\) is a set of complex numbers. A rule \(f\) defined on \(D\) which assigns to every \(z\) in \(D\), a complex number \(w\), is called a function \(f\) or mapping \(f\) on \(D\) and we write \(w = f(z)\). Here \(z\) is a complex variable and can be written as \(z = x + iy\) where \(x, y\), are real and \(i = \sqrt{-1}\).

- The set \(D\) is called domain of definition of \(f\). The set of all \(w = f(z)\) where \(z \in D\) is called the range of \(f\).

- The image of \(z\) under the function \(f\) is \(w = f(z)\) and as stated, this is also a complex number.

- We write \(w = f(z) = u + iv\) where \(u\) and \(v\) are real.

  where \(u\) and \(v\) are called the real and imaginary parts of \(w = f(z)\).

### Analytic Functions:

- **Definition:** Let a function \(f(z)\) be derivable at every point \(z\) in an \(\epsilon\) neighborhood of \(z_0\). i.e., \(f'(z)\) exists for all \(z\) such that \(|z - z_0| < \epsilon\) where \(\epsilon > 0\).

  Then, \(f(z)\) is said to be analytic at \(z_0\).

### Cauchy – Reimann (C-R) Equations:

\[
\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}
\]
Harmonic Functions: \[ \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0 \]

**Cauchy’s Theorem:** Let \( f(z) = u(x, y) + iv(x, y) \) be analytic on and within a simple contour \( c \) and let \( f'(z) \) be continuous there.

Then \[ \int_c f(z) \, dz = 0 \]

**Generalization of Cauchy’s Integral Formula:**
If \( f(z) \) is analytic on and within a simple closed curve \( c \) and if \( a \) is any point with in \( c \), then

\[ f^n(a) = \frac{n!}{2\pi i} \int_c \frac{f(z)}{(z-a)^{n+1}} \, dz \]

**Taylor’s Theorem:**
Let \( f(z) \) be analytic at all points within a circle \( c_0 \) with centre at \( a \) and radius \( r \). Then at each point \( z \) with in \( c_0 \). Then

\[ f(z) = f(a) + f'(a)(z-a) + \frac{f''(a)}{2!}(z-a)^2 + \ldots \]

\[ + \frac{f^n(a)}{n!}(z-a)^n + \ldots \]

**Laurent’s Theorem:** Let \( c_1 \) and \( c_2 \) be two circles given by \( |z' - a| = r_1 \) and \( |z' - a| = r_2 \) respectively where \( r_2 < r_1 \) and \( z' \) is any point on \( c_1 \) or \( c_2 \).

Let \( f(z) \) be analytic on \( c_1 \) and \( c_2 \) and throughout the region between the two circles. Let \( z \) be any point in the ring shaped region between the two circles \( c_1 \) and \( c_2 \).

Then \[ f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z-a)^n} \]

\[ \ldots \ldots \ldots \ldots (1) \]

When \[ a_n = \frac{1}{2\pi i} \oint_{c_1} \frac{f(z')}{(z'-a)^{n+1}} \, dz' \]

\[ b_n = \frac{1}{2\pi i} \oint_{c_2} \frac{f(z')}{(z'-a)^{-n}} \, dz' \]

\[ \text{Where the integrals are taken around } c_1 \text{ and } c_2 \text{ in the anti clock wise sense. [Then series in (1) is called the Laurent’s series expansion of } f(z) \text{ around } z = z_0] \]

**The Calculus Of Residues**

Residues: \[ \oint_c f(z) \, dz = 2\pi i \sum \text{Res } f(z) \]

where \( c \) is a closed curve containing the point \( z = a \) (and such that \( f \) is analytic within and on \( c \))

**Note:** In the above series, the second part

\[ \sum_{n=1}^{\infty} a_{-n} (z-z_0)^{-n} \]

is called the principal part of \( f(z) \) at the singularity \( z = z_0 \).

**Cauchy’s Residue Theorem:**
If \( f(z) \) analytic within and on a closed curve \( c \), except at a finite number of poles \( z_1, z_2, z_3, \ldots, z_n \) within \( c \) and \( R_1, R_2, \ldots, R_n \) be the residues of \( f(z) \) at these poles, then

\[ \oint_c f(z) \, dz = 2\pi i (R_1 + R_2 + \ldots + R_n) \]
If $f(z)$ is analytic within a curve $c$ and has a pole of order $m$ at $z = z_0$, then the residue at $z = z_0$ is

$$\lim_{z \to z_0} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \left[ (z - z_0)^m f(z) \right]$$

Residue at Infinity: $\text{Res} (f : z = \infty) = \frac{1}{2\pi i} \int_{c} f(z) \, dz$.

### Probability & Statistics

#### Probability

- Let $E$ be an event of the experiment. If $m$ elementary events are favorable to an event $E$, then the probability (chance) of $E$ is defined as $P(E) = \frac{m}{n}$.

- Sometimes, $P(E)$ is called the probability of success and $P(\overline{E})$ is called the probability of failure.

- $P(E) = 1 \iff E$ is called a certain event (sure) event.

- $P(E) = 0 \iff E$ is called an impossible event.

**Mutually Exclusive events:** $E_1 \cap E_2 = \phi$

**Complementary Events:**

- $E_1 \cup E_2 = S$ and $E_1 \cap E_2 = \phi$. Then $E_1$, $E_2$ are called complementary events.

#### Conditional Probability:

- $E_1 \subseteq S$, $E_2 \subseteq S$ and $P(E_1) \neq 0$. Then the probability of $E_2$ after the event $E_1$ has occurred is called the conditional probability of $E_2$ given $E_1$ and is denoted by $P\left( \frac{E_2}{E_1} \right)$.

- We define $P\left( \frac{E_2}{E_1} \right) = \frac{P(E_1 \cap E_2)}{P(E_1)}$.

- Similarly we define $P\left( \frac{E_2}{E_1} \right) = \frac{P(E_1 \cap E_2)}{P(E_2)}$.

- Also $P\left( \frac{E_2}{E_1} \right) + P\left( \frac{E_2}{E_1} \right) = 1$.

**Note:** If $E$ is an event of a sample space $S$, then the odds in favour of $E$ are defined as $P(E) P(\overline{E})$ and the odds against $E$ are defined as $P(\overline{E}) : P(E)$.
If \( P(E) : P(\overline{E}) = m : n \), then \( p(E) = \frac{m}{m+n} \) and \( P(\overline{E}) = \frac{n}{m+n} \)

- **Independent and dependent events:**
  - Two events are said to be independent, if one is not depending to another.
  - The events A & B are independent, if \( P(A \cap B) = P(A) \cdot P(B) \)
  - Let \( E_1 \) and \( E_2 \) are two events. The conditional probability of \( E_1 \) given \( E_2 \) is denoted by \( P\left(\frac{E_1}{E_2}\right) \) and is defined by \( P(E_1 \cap E_2)/P(E_2) \), \( P(E_2)\neq0 \).
  - Similarly \( P\left(\frac{E_2}{E_1}\right) = P(E_1 \cap E_2)/P(E_1), \ P(E_1)\neq0 \).
  - If two events are not independent then those are called dependent.

- **Random Variable - Distributions**
  - Random variable:
    - Let \( S \) be a sample space associated with a given random experiment. A real valued function \( X \) which assigns to each element \( s \in S \) one and only one real number \( X(s) = x \) is called a random variable
    - Random Variables are Two Types:
      1. Discrete Random Variable
      2. Continuous Random Variable.

- Mean, Variance of the Random variable (X):
  - Mean = \( \mu \) or \( \bar{x} = \sum x_i \cdot P(X = x_i), if \ exists \)
  - Variance = \( \sigma^2 \sum (x_i - \mu)^2 \cdot P(X = x_i), if \ exists. \)
    \[ \sigma^2 = \sum x_i^2 \cdot P(X = x_i) - \mu^2 \]
    \[ \text{Standard deviation} = \sigma = \sqrt{\text{variance}} \]

  Theoretical Distributions of a discrete random variable

  **Binomial Distribution:**
  
  (Discovered by J. Bernoulli): \( B(x,n,p) = P(X = x) = ^nC_x \cdot q^{n-x} \cdot p^x \)
  
  Where \( p = \) prob. Of success and \( q = \) prob. Of failure. for a binomial variate \( X \), Mean = \( \bar{x} = np \), Variance = \( \sigma^2 = npq \), Standard deviation = \( \sigma = \sqrt{\sigma^2} = \sqrt{npq} \)

  **Poisson distribution:**
  
  \( p(k) = P(X = k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda}, \ (k = 0,1,2,-\ldots) \)
  
  Mean = \( \mu = \lambda \), Variance = \( \sigma^2 = \lambda \), standard deviation = \( \sigma = \sqrt{\lambda} \).

- **STATISTICS**
  - **CORRELATION:**
    
    In a divariate distribution, if the classes in one variable are associated by classes in the other, then variables are called CORRELATED. Correlation is called positive otherwise negative. If the ration of two variable deviation is constant, then correlation is said to be perfect.
Co-efficient of Correlation:

The numerical measure of correlation is called the co-efficient of correlation and is defined as

\[ r = \frac{\sum XY}{n\sigma_x \sigma_y} = \frac{\sum XY}{\sqrt{\sum X^2 \sum Y^2}} \]

\[ \left( \sigma_x^2 = \frac{\sum X^2}{n}; \quad \sigma_y^2 = \frac{\sum Y^2}{n} \right) \]

Where,  
\( X = \text{deviation from mean, } \bar{x} = x - \bar{x} \)
\( Y = \text{deviation from mean, } \bar{y} = y - \bar{y} \)
\( \sigma_x = \text{S. D of x series} \)
\( \sigma_y = \text{S. D of y series} \)
\( N = \text{number of values of the two variables} \)

Methods of Calculation:

a) Direct Method:

Substituting the value of \( \sigma_x \) and \( \sigma_y \) in the above formula, we get

\[ r = \frac{\sum XY}{\sqrt{\sum X^2 \sum Y^2}} \]

\[ \frac{n \sum XY - \sum X \sum Y}{\sqrt{n \sum X^2 - (\sum X)^2} \cdot 4 \times \left( n \sum Y^2 - \sum X^2 \right)} \]

LINE OF REGRESSION:

- If the reactor diagram indicates some relationship between two variables \( X \) and \( Y \), then the dots of the reactor diagram will be concentrated round a curve. This curve is called the curve of regression. When the curve is straight line, then it is called line of regression.

- Equation of the line of regression is,

\[ Y - \bar{Y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x}) \]

And, equation of the line of regression of \( x \) on \( y \) is

\[ x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (Y - \bar{Y}) \]

REGRESSION CO-EFFICIENT:

- Regression co-efficient is defined as the hope of lines of regression, i.e., regression coefficient of line \( x \) on \( y \) = \( r \frac{\sigma_x}{\sigma_y} \)

- In symmetrical distribution, the mean, median and mode coincide but in other distribution they are connected as.

- Mean – Mode = 3(Mean – Median)

Numerical Methods

Numerical Solutions Algebraic Equations:

Given an equation \( f(x) = 0 \)

A solution of \( f(x) = 0 \), is a number \( x = s; \ f(s) = 0 \).
Iteration Method:
To solve \( f(x) = 0 \), when there is no formula for the exact solution, one can use approximation method, an iteration method, in it one start from an initial guess \( x_0 \) (which may be poor) and compute step by step(i.e., searching better) approximation \( x_0, x_1, x_2, \ldots \)of an unknown solution of \( f(x) = 0 \).

Newton–Rapson Method:
\( f(x) = 0 \), where \( f \) is assumed to have a continuous derivative \( f'(x) \).

\[
\begin{align*}
J &= \int f(x) \, dx = \int_a^b f(x) \, dx = x_0^0 + nh \\
&= \frac{h}{2} \left[ (y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1}) \right], \\
h &= \frac{b-a}{n}, \text{ where } x_0 = a \text{ and } x_0 + nh
\end{align*}
\]

Secant Method (Regulafalsi):
\[
x_{n+1} = x_n - \frac{f(x_n) x_{n-1}}{f(x_n) - f(x_{n-1})}
\]

Bisection (Binary–search) Method:
Let \( f \) is continuous function defined on the interval \([a, b]\) with \( f(a) \) and \( f(b) \) of opposite sign. By the intermediate value theorem, there exists a number \( p \) in \((a, b)\) with \( f(p) = 0 \).

[Note: This procedure will work for the case when \( f(a) \) and \( f(b) \) have opposite signs and there is more than one root in the interval \((a, b)\)]

Numerical Solution Of Ordinary Differential Equation

Taylor’s series method:
\[
y' = \frac{dy}{dx} = f(x, y), \text{ with } y(x_0) = y_0
\]

The series about a point \( x = x_0 \)
\[
y = y_0 + (x - x_0)(y')_0 + \frac{(x - x_0)^2}{2!} (y'')_0 + \frac{(x - x_0)^3}{3!} (y''')_0 + \ldots \ldots
\]

Picard’s method:
\[
y' = \frac{dy}{dx} = f(x, y), \text{ with } y(x_0) = y_0 \text{ and}
\]
\[
y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1}) \, dx
\]
Euler’s method:
\[
y' = \frac{dy}{dx} = f(x, y), \quad \text{with} \quad y(x_0) = y_0 \quad \text{and} \quad y_{n+1} = y_n + hf(x_n, y_n)
\]
Where \( h = \frac{x_n - x_0}{n} \) (i.e., \( x_n = x_0 + nh \)).

Runge – Kutta method:

Given initial value problem
\[
y' = \frac{dy}{dx} = f(x, y), \quad \text{where} \quad y(x_0) = y_0
\]
\[
k_1 = hf(x_n, y_n)
\]
\[
k_2 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1)
\]
\[
k_3 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2)
\]
\[
k_4 = hf(x_n + h, y_n + k_3) \quad \text{and} \quad y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)
\]
Where, \( h = \frac{x_n - x_0}{n} \) (i.e., \( x_n = x_0 + nh \)).

FOURIER SINE AND FOURIER COSINE INTEGRAL:

Recalling \( \cos(A - B) \) expansion, expanding the \( \cos(pt - x) \), the Fourier integral of \( f(x) \) given by (4) may be written as,
\[
f(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \left[ \cos(pt) \cos(px) + \sin(pt) \sin(px) \right] f(t) dt dp
\]
\[
\int_{-\infty}^{\infty} \cos(px) \int_{-\infty}^{\infty} \cos(pt) f(t) dt dp + \int_{-\infty}^{\infty} \sin(px) \int_{-\infty}^{\infty} \sin(pt) f(t) dt dp
\]

When \( f(t) \) is an odd function \( \cos pt f(t) \) is an odd function and \( \sin pt f(t) \) is an even function. So, the first integral in the right side of (1) becomes zero. Therefore, we get
\[
f(x) = \frac{2}{\pi} \int_{0}^{\infty} \sin px \int_{0}^{\infty} f(t) \sin pt dt dp
\]

Which is known as the Fourier sine integral.
In the same way, when \( f(t) \) is an even function, the second integral in the right side of (2) becomes zero. Therefore, we get

\[
f(x) = \frac{2}{\pi} \int_0^\infty \cos px \int_0^\infty f(t) \cos pt \, dt \, dp
\]

Which is known as the **Fourier cosine integral**.

**LAPLACE TRANSFORMS**

**Definition:**

\[
L \{ f(t) \} = \mathcal{F}(s) = \int_0^\infty e^{-st} f(t) \, dt
\]

Provided that the integral exists. Here the parameter \( s \) is a real or complex number.

The relation (1) can also be written as \( f(t) = L^{-1} \{ \mathcal{F}(s) \} \)

In such a case, the function \( f(t) \) is said to be the inverse Laplace transform of \( \mathcal{F}(s) \). The symbol \( L \) which transforms \( f(t) \) into \( \mathcal{F}(s) \) can be called the inverse Laplace transform operator.

**Laplace Transforms of Elementary Functions:**

1. \( L\{1\} = \frac{1}{s} \)
2. \( L\{t\} = \frac{1}{s^2} \)
3. \( L\{t^n\} = \frac{n!}{s^{n+1}} \) where \( n \) is a positive integer
   \[ = \frac{\Gamma(n+1)}{s^{n+1}}, \text{otherwise} \]
4. \( L\{e^{at}\} = \frac{1}{s-a}, \text{if } s>a \)
5. \( L\{\sinh at\} = \frac{a}{s^2-a^2}, \text{if } s>|a| \)
6. \( L\{\cosh at\} = \frac{s}{s^2-a^2}, \text{Re}(s)>a \)
7. \( L\{\sin at\} = \frac{a}{s^2+a^2}, \text{if } s>0 \)
8. \( L\{\cos at\} = \frac{s}{s^2+a^2}, \text{if } s>0 \)

**Z – TRANSFORMS**

**Definition:**

\[ Z\{f(n)\} = \sum_{n=0}^{\infty} f(n)z^{-n}, \text{ for } n=0,1,2,\ldots\]\n
the right hand side series is convergent, we write \( Z\{f(n)\} = F(z) \)

The inverse Z transform of \( F(z) \) is written as \( Z^{-1}\{F(z)\} = f(n) \) whenever \( Z\{f(n)\} = F(z) \)

**Z Transforms of Some Standard Functions**

1. \( Z(1) = \frac{z}{z-1} \)
2. \( Z\{a^n\} = \frac{z}{z-a} \)
3. \( Z\{(-1)^n\} = \frac{z}{z-(-1)} = \frac{z}{z+1} \)
4. \( Z(n) = \frac{z}{(z-1)^2} \)
5. \( Z(n^n) = -z \frac{d}{dz} [Z(n^{n-1})] \)
6. \( Z(n) = \frac{z}{(z-1)^2} \)
7. \[ Z(n^2) = \frac{z^2 + z}{(z-1)^2} \]

8. \[ Z\left(n^a\right) = \frac{az}{(z-a)^2} \]

9. \[ Z\left(\frac{1}{n!}\right) = e^{\frac{1}{z}} \]

\[ \therefore e^z = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \]

10. \[ Z\left(\frac{1}{n}\right) = \log \frac{z}{z-1} \]

11. \[ Z\left(\frac{1}{n+1}\right) = z \cdot \log \left(\frac{z}{z-1}\right) \]

12. \( \mathcal{Z} \) - transform of discrete unit step function:

13. Unit step sequence \( f(n) \) is defined by \( f(n) = 0 \) for \( n < 0 \), \( f(n) = 1 \) for \( n \geq 0 \).

\[ \therefore Z[f(n)] = \sum_{n=0}^{\infty} f(n)z^{-n} = \sum_{n=0}^{\infty} z^{-n} \]

\[ = 1 + \frac{1}{z} + \frac{1}{z^2} + \ldots + \frac{1}{z^n} + \ldots = \frac{1}{1 - \frac{1}{z}} = \frac{z}{z-1} \]

(This is in G.P. its sum is \( \frac{a}{1-r} \))

\[ \mathbf{Properties \ of \ Z-Transforms:} \]

\textbf{Linearity Property:}
If \( a, b \) are any constants and \( f(n) \) and \( g(n) \) be any discrete functions, then \( Z[af(n) + bg(n)] = aZ[f(n)] + bZ[g(n)] \)

\textbf{Change of Scale property (or Damping Rule):}

i) If \( Z[f(n)] = F(z) \), then \( Z[a^{-n}f(n)] = F(az) \)

ii) If \( \mathcal{Z}[f(n)] = F(z) \), then \( \mathcal{Z}[a^n f(n)] = F\left(\frac{z}{a}\right) \)
Space for Rough Work
Space for Rough Work